# "XRNG" RANDOM NUMBER GENERATOR

## PROJECT TEST REPORT

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**ABSTRACT:** Empirical evidence of XRNG method is presented. XRNG passes all tests for randomness specified by Federal Information Processing Standards Publication 140-1 and Diehard test battery.

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#### 1.0 Introduction

This document provides test result data for the XRNG true-random number generation method to demonstrate empirical evidence of the method to interested parties outside RNG Research. The scope of this document is limited to Federal Information Processing Standards Publication 140-1 (FIPS PUB 140-1)<sup>1</sup> and Diehard<sup>2</sup> test results. XRNG apparatus have passed all tests for randomness to which they have been subjected.

#### 2.0 Background

XRNG is a robust method of generating truly random sequences of numbers which fall into the discrete uniform distribution over the desired interval. This is accomplished by measuring the output of a truly random noise source (e.g., semiconductor noise) with an analog-to-digital (A/D) converter, and applying a modular reduction to the measurements. Assuming the normal distribution for the noise, the theory of operation predicts maximum biases (expressed as the difference between the computed probability of an XRNG outputting a given number and the probability based on an ideal uniform distribution) for two sets of conditions as follows:

A/D Converter Resolution	Standard Deviation of A/D Converter Output Distribution	Reduction Modulus	Theoretical Bias is less than
16 bits	4,096 codes	256	$3 \times 10^{-12}$
8 bits	16 codes	16	$4 \times 10^{-9}$

#### 3.0 Procedure

Truly random numbers modulo-256 were generated by XRNG apparatus comprising a reverse-biased P-N junction semiconductor noise source, 16-bit A/D converter, and interface means by which data bits D8 through D15 were discarded. Four (4) files of 32,768 bytes each were generated and the first 20,000 bits (i.e., the first 2,500 bytes) of each were subjected to the tests for randomness specified in FIPS PUB 140-1. Eight (8) files of 10,485,760 bytes each were also generated and subjected to all fifteen (15) tests in the MS-DOS version of the Diehard test suite. The first 20,000 bits of each of these files were also subjected to the FIPS PUB 140-1 tests as above.

Lastly, random numbers modulo-16 were generated by XRNG apparatus comprising a reverse-biased P-N junction semiconductor noise source, 8-bit A/D converter and interface means by which data bits D4 through D7 were discarded. Odd nybbles in a random sequence of 20,971,520 nybbles were concatenated with the immediately-following even nybble to form a sequence of 10,485,760 bytes, which was then subjected to the Diehard test suite.

#### 4.0 Results

XRNG apparatus successfully passed all tests for randomness. No deviation from the ideal uniform distribution was detected in the XRNG data.

Observed Bias	none	none											
Tests Run F=FIPS D=Diehard	ĹIJ	Ĺц	ы	ſц	F, D	F,D	F,D	F, D	F, D	F,D	F, D	F, D	۵
Theoretical Bias is less than	$3 \times 10^{-12}$	$4 \times 10^{-9}$											
Reduction Modulus	256	256	256	256	256	256	256	256	256	256	256	256	16
Standard Deviation of A/D Converter Output Distribution	4,300 codes	16 codes											
A/D Converter Resolution	16 bits	8 bits											
file	sample1.rng (32K)	sample2.rng (32K)	sample3.rng (32K)	sample4.rng (32K)	block0.rng (10M)	blockl.rng (10M)	block2.rng (10M)	block3.rng (10M)	block4.rng (10M)	block5.rng (10M)	block6.rng (10M)	block7.rng (10M)	blockx.rng (10M)

TESTS FOR RANDOMNESS ON RANDOM SEQUENCES GEREATED BY XRNG METHOD

Test results are tabulated as follows:

Output reports generated by the program running the FIPS PUB 140-1 tests are presented in Appendix A. The full text of the Diehard output files are included in Appendix B.

#### 5.0 Conclusions

Data produced by XRNG method has passed all tests for randomness to which it has been subjected. No bias has been detected. Empirical evidence of the method is thus presented.

#### 6.0 References

- 1. United States Government Commerce Department, Ronald H. Brown, Secretary. National Institute of Standards and Technology. (1994) Federal Information Processing Standards Publication 140-1: Security Requirements for Cryptographic Modules (see http://csrc.ncsl.nist.gov/fips/fips1401.htm)
- Marsaglia, G. (1997) <u>Diehard: a battery of tests for random number</u> <u>generators</u>. published online on Florida State University Department of Statistics web site at http://stat.fsu.edu/~geo/diehard.html.

Results of FIPS 140-1 Specified Tests on sample1.rng BLOCK 1 Monobit X= 9936 PASS (pass if 9654 < X < 10346) Poker X= 9.7728 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2536 2547 pass (pass if 2267 < X < 2733) 2 1255 1258 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 597 640 4 336 287 pass (pass if 223 < X < 402) 5 157 pass (pass if 90 < X < 223) 147 6+ 165 147 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X = 10066PASS (pass if 9654 < X < 10346) Poker X= 9.0048 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2481 2452 pass (pass if 2267 < X < 2733) 2 1254 1205 pass (pass if 1079 < X < 1421) 3 616 670 pass (pass if 502 < X < 748) 4 315 pass (pass if 223 < X < 402) 314 5 153 175 pass (pass if 90 < X < 223) 6+ 153 153 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10032 PASS (pass if 9654 < X < 10346) Poker X= 16.0832 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2444 2490 pass (pass if 2267 < X < 2733) 1 2 1268 1220 pass (pass if 1079 < X < 1421) 3 639 613 pass (pass if 502 < X < 748) 4 312 310 pass (pass if 223 < X < 402) 5 144 173 pass (pass if 90 < X < 223) 6+ 160 160 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9957 PASS (pass if 9654 < X < 10346) Poker X= 9.6384 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2487 2547 pass (pass if 2267 < X < 2733) 1 2 1267 1228 pass (pass if 1079 < X < 1421) 3 590 601 pass (pass if 502 < X < 748) 4 356 291 pass (pass if 223 < X < 402) 5 161 183 pass (pass if 90 < X < 223) 6+ 139 151 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

Appendix A: FIPS PUB 140-1 Test Result Details

BLOCK 5 Monobit X= 10134 PASS (pass if 9654 < X < 10346) Poker X= 22.3424 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2483 2439 pass (pass if 2267 < X < 2733) 2 1240 1247 pass (pass if 1079 < X < 1421) 3 588 pass (pass if 502 < X < 748) 624 4 326 299 pass (pass if 223 < X < 402) 5 pass (pass if 90 < X < 223) 148 169 б+ 161 186 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on sample2.rng BLOCK 1 Monobit X= 10010 PASS (pass if 9654 < X < 10346) Poker X= 11.008 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2519 2530 pass (pass if 2267 < X < 2733) 2 1210 1218 pass (pass if 1079 < X < 1421) 644 pass (pass if 502 < X < 748) 3 651 pass (pass if 223 < X < 402) 4 308 322 pass (pass if 90 < X < 223) 5 158 138 6+ 145 166 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9998 PASS (pass if 9654 < X < 10346) Poker X= 11.6928 PASS (pass if 1.03 < X < 57.4) Ones Run length Zeros 2479 2476 pass (pass if 2267 < X < 2733) 1 1274 pass (pass if 1079 < X < 1421) 2 1255 pass (pass if 502 < X < 748) 3 654 636 pass (pass if 223 < X < 402) 4 321 325 5 136 139 pass (pass if 90 < X < 223) 6+ 159 155 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10003 PASS (pass if 9654 < X < 10346) Poker X= 19.4944 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2497 2502 pass (pass if 2267 < X < 2733) 1 pass (pass if 1079 < X < 1421) 2 1247 1314 pass (pass if 502 < X < 748) 3 584 638 pass (pass if 223 < X < 402) 4 282 312 5 165 151 pass (pass if 90 < X < 223) 6+ 164 157 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

BLOCK 4 Monobit X= 9832 PASS (pass if 9654 < X < 10346) Poker X= 16.7616 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2399 2486 pass (pass if 2267 < X < 2733) 2 1201 1219 pass (pass if 1079 < X < 1421) 3 639 pass (pass if 502 < X < 748) 676 4 321 313 pass (pass if 223 < X < 402) 5 176 130 pass (pass if 90 < X < 223) б+ 169 154 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 9915 PASS (pass if 9654 < X < 10346) Poker X= 9.664 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2437 2471 pass (pass if 2267 < X < 2733) 1 1255 pass (pass if 1079 < X < 1421) 2 1251 3 627 633 pass (pass if 502 < X < 748) 4 313 327 pass (pass if 223 < X < 402) 5 178 125 pass (pass if 90 < X < 223) б+ 163 158 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on sample3.rng BLOCK 1 Monobit X= 9973 PASS (pass if 9654 < X < 10346) Poker X= 14.4512 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2496 2447 pass (pass if 2267 < X < 2733) 1 1288 pass (pass if 1079 < X < 1421) 2 1218 pass (pass if 502 < X < 748) 3 640 632 pass (pass if 223 < X < 402) 4 317 316 5 151 162 pass (pass if 90 < X < 223) 6+ 164 141 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 10003 PASS (pass if 9654 < X < 10346) Poker X= 20.8576 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2538 2588 pass (pass if 2267 < X < 2733) 1 pass (pass if 1079 < X < 1421) 2 1273 1239 pass (pass if 502 < X < 748) 3 643 619 pass (pass if 223 < X < 402) 4 311 293 5 146 162 pass (pass if 90 < X < 223) 6+ 146 155 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

BLOCK 3 Monobit X= 9962 PASS (pass if 9654 < X < 10346) Poker X= 17.568 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2535 2601 pass (pass if 2267 < X < 2733) 2 1264 1245 pass (pass if 1079 < X < 1421) 3 618 pass (pass if 502 < X < 748) 626 4 339 296 pass (pass if 223 < X < 402) 5 pass (pass if 90 < X < 223) 148 129 pass (pass if 90 < X < 223) б+ 144 167 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 10055 PASS (pass if 9654 < X < 10346) Poker X= 18.592 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2490 2492 pass (pass if 2267 < X < 2733) 1 1181 pass (pass if 1079 < X < 1421) 2 1220 670 pass (pass if 502 < X < 748) 3 627 4 346 300 pass (pass if 223 < X < 402) 5 147 165 pass (pass if 90 < X < 223) б+ 148 170 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 9910 PASS (pass if 9654 < X < 10346) Poker X= 11.296 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2445 2541 pass (pass if 2267 < X < 2733) 1 2 1251 1212 pass (pass if 1079 < X < 1421) 3 649 603 pass (pass if 502 < X < 748) 4 320 307 pass (pass if 223 < X < 402) 154 pass (pass if 90 < X < 223) 5 140 6+ 176 165 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

Results of FIPS 140-1 Specified Tests on sample4.rng

BLOCK 1

ss)
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BLOCK 2 Monobit X= 9957 PASS (pass if 9654 < X < 10346) Poker X= 10.88 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2514 2522 pass (pass if 2267 < X < 2733) 2 1260 1243 pass (pass if 1079 < X < 1421) 3 621 pass (pass if 502 < X < 748) 593 306 4 320 pass (pass if 223 < X < 402) 5 pass (pass if 90 < X < 223) 165 155 pass (pass if 90 < X < 223) 6+ 173 149 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 9949 PASS (pass if 9654 < X < 10346) Poker X= 18.2784 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2536 2532 pass (pass if 2267 < X < 2733) 1 1298 pass (pass if 1079 < X < 1421) 2 1268 3 640 623 pass (pass if 502 < X < 748) 4 279 291 pass (pass if 223 < X < 402) 5 156 157 pass (pass if 90 < X < 223) б+ 166 144 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9950 PASS (pass if 9654 < X < 10346) Poker X= 12.0192 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2471 1 2484 pass (pass if 2267 < X < 2733) 2 1256 1276 pass (pass if 1079 < X < 1421) 3 618 657 pass (pass if 502 < X < 748) 4 304 290 pass (pass if 223 < X < 402) 146 pass (pass if 90 < X < 223) 5 161 6+ 167 150 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 10018 PASS (pass if 9654 < X < 10346) Poker X= 21.728 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2586 2481 pass (pass if 2267 < X < 2733) 1 2 1208 1279 pass (pass if 1079 < X < 1421) 3 589 665 pass (pass if 502 < X < 748) 4 313 304 pass (pass if 223 < X < 402) 5 171 154 pass (pass if 90 < X < 223) 144 pass (pass if 90 < X < 223) 6+ 160 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

Results of FIPS 140-1 Specified Tests on block0.rng BLOCK 1 Monobit X= 9915 PASS (pass if 9654 < X < 10346) Poker X= 13.664 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2558 pass (pass if 2267 < X < 2733) 1 2506 2 1239 1245 pass (pass if 1079 < X < 1421) 3 644 629 pass (pass if 502 < X < 748) 4 316 286 pass (pass if 223 < X < 402) 5 165 151 pass (pass if 90 < X < 223) 6+ 154 156 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9908 PASS (pass if 9654 < X < 10346) Poker X= 19.9808 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2478 2570 pass (pass if 2267 < X < 2733) 1 2 1262 1289 pass (pass if 1079 < X < 1421) 3 690 559 pass (pass if 502 < X < 748) 4 298 299 pass (pass if 223 < X < 402) 5 154 154 pass (pass if 90 < X < 223) 6+ 163 pass (pass if 90 < X < 223) 152 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10003 PASS (pass if 9654 < X < 10346) Poker X= 15.9232 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2464 2497 pass (pass if 2267 < X < 2733) 2 1271 1235 pass (pass if 1079 < X < 1421) 3 635 647 pass (pass if 502 < X < 748) 4 289 pass (pass if 223 < X < 402) 313 5 169 pass (pass if 90 < X < 223) 145 151 pass (pass if 90 < X < 223) 6+ 159 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9924 PASS (pass if 9654 < X < 10346) Poker X= 8.3264 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2497 2556 pass (pass if 2267 < X < 2733) 2 1252 1201 pass (pass if 1079 < X < 1421) 634 pass (pass if 502 < X < 748) 3 601 pass (pass if 223 < X < 402) 4 319 295 pass (pass if 90 < X < 223) 5 163 161 pass (pass if 90 < X < 223) 6+ 168 152 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 PASS (pass if 9654 < X < 10346) Monobit X= 9989 Poker X= 21.5488 PASS (pass if 1.03 < X < 57.4)

Run length Zeros Ones 1 2458 2491 pass (pass if 2267 < X < 2733) 1220 pass (pass if 1079 < X < 1421) 2 1228 3 634 627 pass (pass if 502 < X < 748) 311 pass (pass if 223 < X < 402) 4 341 5 156 150 pass (pass if 90 < X < 223) 168 pass (pass if 90 < X < 223) 6+ 151 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block1.rng BLOCK 1 Monobit X= 10046 PASS (pass if 9654 < X < 10346) Poker X= 8.8384 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2515 2473 pass (pass if 2267 < X < 2733) 1 2 1222 1270 pass (pass if 1079 < X < 1421) 3 639 594 pass (pass if 502 < X < 748) 4 318 318 pass (pass if 223 < X < 402) 5 140 173 pass (pass if 90 < X < 223) 6+ 159 pass (pass if 90 < X < 223) 153 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9966 PASS (pass if 9654 < X < 10346) PASS (pass if 1.03 < X < 57.4) Poker X= 15.2576 Run length Zeros Ones 1 2455 2447 pass (pass if 2267 < X < 2733) 2 1189 1192 pass (pass if 1079 < X < 1421) 3 603 642 pass (pass if 502 < X < 748) 4 343 309 pass (pass if 223 < X < 402) 5 164 pass (pass if 90 < X < 223) 158 pass (pass if 90 < X < 223) 6+ 174 167 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 9950 PASS (pass if 9654 < X < 10346) Poker X= 19.0464 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2516 2551 pass (pass if 2267 < X < 2733) 2 1265 1230 pass (pass if 1079 < X < 1421) 638 pass (pass if 502 < X < 748) 3 625 pass (pass if 223 < X < 402) 4 271 297 pass (pass if 90 < X < 223) 5 165 151 pass (pass if 90 < X < 223) 6+ 178 153 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 PASS (pass if 9654 < X < 10346) Monobit X= 9923 Poker X= 18.4896 PASS (pass if 1.03 < X < 57.4)

Run length Zeros Ones 1 2431 2496 pass (pass if 2267 < X < 2733) pass (pass if 1079 < X < 1421) 2 1289 1232 3 608 645 pass (pass if 502 < X < 748) 4 307 291 pass (pass if 223 < X < 402) 5 173 148 pass (pass if 90 < X < 223) 6+ 157 pass (pass if 90 < X < 223) 161 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 10119 PASS (pass if 9654 < X < 10346) Poker X= 19.168 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2525 2465 pass (pass if 2267 < X < 2733) 2 1254 1274 pass (pass if 1079 < X < 1421) 597 pass (pass if 502 < X < 748) 3 611 4 302 331 pass (pass if 223 < X < 402) 5 172 pass (pass if 90 < X < 223) 160 б+ 143 156 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block2.rng BLOCK 1 Monobit X= 9980 PASS (pass if 9654 < X < 10346) PASS (pass if 1.03 < X < 57.4) Poker X= 12.1472 Run length Zeros Ones 1 2461 2468 pass (pass if 2267 < X < 2733) 2 1234 1230 pass (pass if 1079 < X < 1421) 3 617 661 pass (pass if 502 < X < 748) 4 326 290 pass (pass if 223 < X < 402) 5 135 pass (pass if 90 < X < 223) 140 pass (pass if 90 < X < 223) 6+ 180 175 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9941 PASS (pass if 9654 < X < 10346) Poker X= 13.472 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones pass (pass if 2267 < X < 2733) 1 2547 2551 2 1178 1199 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 651 638 pass (pass if 223 < X < 402) 4 315 335 pass (pass if 90 < X < 223) 5 154 148 pass (pass if 90 < X < 223) 6+ 171 145 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 PASS (pass if 9654 < X < 10346) Monobit X= 9941 Poker X= 19.8336 PASS (pass if 1.03 < X < 57.4)

Run length Zeros Ones 1 2539 2559 pass (pass if 2267 < X < 2733) 2 1202 1203 pass (pass if 1079 < X < 1421) 3 599 606 pass (pass if 502 < X < 748) 4 344 320 pass (pass if 223 < X < 402) 5 163 164 pass (pass if 90 < X < 223) 159 154 pass (pass if 90 < X < 223) 6+ PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9974 PASS (pass if 9654 < X < 10346) Poker X= 8.9472 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2523 2542 pass (pass if 2267 < X < 2733) 2 1312 1307 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 588 608 4 334 305 pass (pass if 223 < X < 402) 5 151 142 pass (pass if 90 < X < 223) pass (pass if 90 < X < 223) б+ 151 156 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 9998 PASS (pass if 9654 < X < 10346) Poker X= 6.7456 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2477 2463 pass (pass if 2267 < X < 2733) 2 1249 1257 pass (pass if 1079 < X < 1421) 3 600 631 pass (pass if 502 < X < 748) 4 335 319 pass (pass if 223 < X < 402) 5 175 152 pass (pass if 90 < X < 223) 6+ 143 158 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block3.rng BLOCK 1 Monobit X= 10040 PASS (pass if 9654 < X < 10346) Poker X= 9.7408 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones pass (pass if 2267 < X < 2733) 1 2487 2519 2 1265 1227 pass (pass if 1079 < X < 1421) 613 pass (pass if 502 < X < 748) 3 659 pass (pass if 223 < X < 402) 4 288 319 5 pass (pass if 90 < X < 223) 159 145 pass (pass if 90 < X < 223) 6+ 159 166 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 10017 PASS (pass if 9654 < X < 10346) Poker X= 15.6032 PASS (pass if 1.03 < X < 57.4)

Run length Zeros Ones 1 2484 2478 pass (pass if 2267 < X < 2733) 1224 pass (pass if 1079 < X < 1421) 2 1249 3 621 647 pass (pass if 502 < X < 748) 4 278 281 pass (pass if 223 < X < 402) 5 165 173 pass (pass if 90 < X < 223) 6+ 170 165 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10091 PASS (pass if 9654 < X < 10346) Poker X= 7.9232 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2560 2473 pass (pass if 2267 < X < 2733) 2 1220 1265 pass (pass if 1079 < X < 1421) 654 pass (pass if 502 < X < 748) 3 624 4 320 312 pass (pass if 223 < X < 402) 5 152 pass (pass if 90 < X < 223) 133 161 pass (pass if 90 < X < 223) б+ 161 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9992 PASS (pass if 9654 < X < 10346) Poker X= 9.8752 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2463 2444 pass (pass if 2267 < X < 2733) 1286 pass (pass if 1079 < X < 1421) 2 1243 3 644 603 pass (pass if 502 < X < 748) 4 298 332 pass (pass if 223 < X < 402) 5 158 151 pass (pass if 90 < X < 223) 6+ 164 154 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 10023 PASS (pass if 9654 < X < 10346) Poker X= 11.04 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2526 2475 pass (pass if 2267 < X < 2733) 2 1272 1274 pass (pass if 1079 < X < 1421) 3 608 639 pass (pass if 502 < X < 748) 4 302 333 pass (pass if 223 < X < 402) 5 139 161 pass (pass if 90 < X < 223) 6+ 171 135 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

Results of FIPS 140-1 Specified Tests on block4.rng

BLOCK 1								
Monobit X= 9993	PASS	(pass	if	9654	<	Х	<	10346)
Poker X= 17.7152	PASS	(pass	if	1.03	<	Х	<	57.4)

Run length Zeros Ones 1 2568 2519 pass (pass if 2267 < X < 2733) 1280 pass (pass if 1079 < X < 1421) 2 1249 3 600 627 pass (pass if 502 < X < 748) pass (pass if 223 < X < 402) 4 314 302 5 142 150 pass (pass if 90 < X < 223) 6+ 158 pass (pass if 90 < X < 223) 163 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9947 PASS (pass if 9654 < X < 10346) Poker X= 4.288 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2503 2492 pass (pass if 2267 < X < 2733) 2 1207 1211 pass (pass if 1079 < X < 1421) 645 pass (pass if 502 < X < 748) 3 582 4 335 309 pass (pass if 223 < X < 402) 5 155 pass (pass if 90 < X < 223) 171 6+ 172 158 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10000 PASS (pass if 9654 < X < 10346) Poker X= 19.68 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2453 2517 pass (pass if 2267 < X < 2733) 2 1299 1212 pass (pass if 1079 < X < 1421) 3 636 626 pass (pass if 502 < X < 748) 4 311 328 pass (pass if 223 < X < 402) 5 156 157 pass (pass if 90 < X < 223) 6+ 146 160 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9984 PASS (pass if 9654 < X < 10346) Poker X= 14.3936 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2465 2495 pass (pass if 2267 < X < 2733) 2 1315 1231 pass (pass if 1079 < X < 1421) 3 594 665 pass (pass if 502 < X < 748) 4 321 296 pass (pass if 223 < X < 402) 5 154 164 pass (pass if 90 < X < 223) 6+ 150 148 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 9974 PASS (pass if 9654 < X < 10346) Poker X= 11.4752 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2483 2452 pass (pass if 2267 < X < 2733) 1 2 1333 pass (pass if 1079 < X < 1421) 1261 3 610 576 pass (pass if 502 < X < 748) 4 330 340 pass (pass if 223 < X < 402)

145 pass (pass if 90 < X < 223) 5 149 pass (pass if 90 < X < 223) б+ 160 147 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block5.rng BLOCK 1 Monobit X= 9970 PASS (pass if 9654 < X < 10346) Poker X= 31.3792 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2501 2522 pass (pass if 2267 < X < 2733) 2 1256 1257 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 635 607 4 311 324 pass (pass if 223 < X < 402) 5 168 pass (pass if 90 < X < 223) 169 6+ 143 136 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9915 PASS (pass if 9654 < X < 10346) Poker X= 16.6656 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2520 2575 pass (pass if 2267 < X < 2733) 2 1253 1198 pass (pass if 1079 < X < 1421) 3 608 629 pass (pass if 502 < X < 748) 4 302 327 pass (pass if 223 < X < 402) 5 153 138 pass (pass if 90 < X < 223) 6+ 182 152 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10034 PASS (pass if 9654 < X < 10346) Poker X= 14.0096 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2518 2491 pass (pass if 2267 < X < 2733) 2 1274 1257 pass (pass if 1079 < X < 1421) 3 587 620 pass (pass if 502 < X < 748) 4 306 322 pass (pass if 223 < X < 402) 5 165 171 pass (pass if 90 < X < 223) 6+ 158 146 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9997 PASS (pass if 9654 < X < 10346) Poker X= 18.5984 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2494 2414 pass (pass if 2267 < X < 2733) 1 2 1206 1294 pass (pass if 1079 < X < 1421) 3 580 588 pass (pass if 502 < X < 748) 4 339 346 pass (pass if 223 < X < 402)

163 pass (pass if 90 < X < 223) 5 171 pass (pass if 90 < X < 223) б+ 165 150 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 9977 PASS (pass if 9654 < X < 10346) Poker X= 34.2144 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2342 2423 pass (pass if 2267 < X < 2733) 1 2 1279 1239 pass (pass if 1079 < X < 1421) 3 647 627 pass (pass if 502 < X < 748) 4 342 303 pass (pass if 223 < X < 402) 165 pass (pass if 90 < X < 223) 5 152 6+ 150 156 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block6.rng BLOCK 1 Monobit X= 10008 PASS (pass if 9654 < X < 10346) Poker X= 16.4928 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2512 2491 pass (pass if 2267 < X < 2733) 2 1246 1233 pass (pass if 1079 < X < 1421) 3 601 624 pass (pass if 502 < X < 748) 349 4 300 pass (pass if 223 < X < 402) 5 158 135 pass (pass if 90 < X < 223) 6+ 168 153 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9970 PASS (pass if 9654 < X < 10346) Poker X= 8.1088 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2540 2521 pass (pass if 2267 < X < 2733) 2 1213 1289 pass (pass if 1079 < X < 1421) 3 647 590 pass (pass if 502 < X < 748) 4 299 285 pass (pass if 223 < X < 402) 5 170 170 pass (pass if 90 < X < 223) б+ 150 164 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 10022 PASS (pass if 9654 < X < 10346) Poker X= 14.7072 PASS (pass if 1.03 < X < 57.4) Ones Run length Zeros 2547 2521 pass (pass if 2267 < X < 2733) 1 2 1241 1256 pass (pass if 1079 < X < 1421) 3 623 610 pass (pass if 502 < X < 748) 4 315 346 pass (pass if 223 < X < 402)

139 pass (pass if 90 < X < 223) 5 138 pass (pass if 90 < X < 223) б+ 165 158 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 9911 PASS (pass if 9654 < X < 10346) Poker X= 17.9264 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2490 2498 pass (pass if 2267 < X < 2733) 1 2 1213 1173 pass (pass if 1079 < X < 1421) 3 614 692 pass (pass if 502 < X < 748) 4 286 302 pass (pass if 223 < X < 402) 158 pass (pass if 90 < X < 223) 5 181 6+ 182 143 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 10036 PASS (pass if 9654 < X < 10346) Poker X= 14.0736 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2534 2471 pass (pass if 2267 < X < 2733) 2 1252 1286 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 619 645 pass (pass if 223 < X < 402) 4 320 353 pass (pass if 90 < X < 223) 5 164 142 6+ 141 133 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) Results of FIPS 140-1 Specified Tests on block7.rng BLOCK 1 Monobit X= 10056 PASS (pass if 9654 < X < 10346) Poker X= 12.2112 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2572 2500 pass (pass if 2267 < X < 2733) 2 1199 1304 pass (pass if 1079 < X < 1421) 3 640 602 pass (pass if 502 < X < 748) 4 322 311 pass (pass if 223 < X < 402) 5 140 140 pass (pass if 90 < X < 223) 6+ 153 168 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 2 Monobit X= 9980 PASS (pass if 9654 < X < 10346) Poker X= 21.9456 PASS (pass if 1.03 < X < 57.4) Ones Run length Zeros pass (pass if 2267 < X < 2733) 2450 2489 1 2 1233 pass (pass if 1079 < X < 1421) 1270 3 643 632 pass (pass if 502 < X < 748) 4 324 314 pass (pass if 223 < X < 402)

172 pass (pass if 90 < X < 223) 5 135 6+ pass (pass if 90 < X < 223) 162 144 PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 3 Monobit X= 9986 PASS (pass if 9654 < X < 10346) Poker X= 24.5824 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 2511 2525 pass (pass if 2267 < X < 2733) 1 2 1289 1278 pass (pass if 1079 < X < 1421) 3 641 640 pass (pass if 502 < X < 748) 4 308 313 pass (pass if 223 < X < 402) 5 152 154 pass (pass if 90 < X < 223) 6+ 147 138 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 4 Monobit X= 10035 PASS (pass if 9654 < X < 10346) Poker X= 18.4704 PASS (pass if 1.03 < X < 57.4) Run length Zeros Ones 1 2576 2538 pass (pass if 2267 < X < 2733) 2 1251 1220 pass (pass if 1079 < X < 1421) pass (pass if 502 < X < 748) 3 606 648 pass (pass if 223 < X < 402) 4 337 316 5 pass (pass if 90 < X < 223) 148 173 6+ 150 131 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run) BLOCK 5 Monobit X= 10029 PASS (pass if 9654 < X < 10346) Poker X= 9.1776 PASS (pass if 1.03 < X < 57.4) Ones Run length Zeros 2438 2472 pass (pass if 2267 < X < 2733) 1 2 1215 pass (pass if 1079 < X < 1421) 1274 pass (pass if 502 < X < 748) 3 641 643 pass (pass if 223 < X < 402) 4 310 322 5 167 154 pass (pass if 90 < X < 223) 6+ 140 164 pass (pass if 90 < X < 223) PASS (pass if all twelve counts pass) No long run (LEN => 34) PASS (pass if no long run)

#### Appendix B: Diehard Test Result Details

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p > .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results ::  $\colon\colon$  to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: :: Each set of bits provides a p-value, and the nine p-values :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block0.rng For a sample of size 500: mean block0.rng using bits 1 to 24 1.916 duplicate number number spacings observed expected 0 67.668 61. 141. 1 135.335 2 135.335 157. 3 90.224 88. 4 45.112 35. 5 11. 18.045 6 to INF 7. 8.282 .858982 Chisquare with 6 d.o.f. = 9.63 p-value= For a sample of size 500: mean block0.rng using bits 2 to 25 1.910 duplicate number number spacings observed expected 0 69. 67.668 1 135.335 142. 2 144. 135.335

3 85. 90.224 4 44. 45.112 18.045 5 7. 9. 6 to INF 8.282 .766366 Chisquare with 6 d.o.f. =8.06 p-value= For a sample of size 500: mean using bits 3 to 26 1.996 block0.rng duplicate number number spacings observed expected 65. 67.668 0 1 145. 135.335 2 130. 135.335 3 89. 90.224 4 47. 45.112 5 14. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. = 2.36 p-value= .116675 For a sample of size 500: mean block0.rng using bits 4 to 27 2.022 duplicate number number spacings observed expected 0 72. 67.668 1 132. 135.335 2 135.335 118. 3 97. 90.224 4 55. 45.112 5 24. 18.045 6 to INF 2. 8.282 Chisquare with 6 d.o.f. = 11.99 p-value= .937734 For a sample of size 500: mean block0.rng using bits 5 to 28 1.960 duplicate number number spacings observed expected 0 69. 67.668 1 140. 135.335 2 136. 135.335 3 83. 90.224 4 49. 45.112 5 15. 18.045 6 to INF 8. 8.282 Chisquare with 6 d.o.f. =1.63 p-value= .049389 For a sample of size 500: mean block0.rng using bits 6 to 29 2.024 duplicate number number spacings observed expected 54. 67.668 0 1 138. 135.335 2 147. 135.335 3 89. 90.224 4 53. 45.112 5 16. 18.045 6 to INF 8.282 3. 8.81 p-value= .815725 Chisquare with 6 d.o.f. =

															•																									
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•

bloc	k0.rng	ample of size using bits		
duplicate	number	number		
spacings	observed	expected	b	
0	72.	67.668		
1	114.	135.335		
2	144.	135.335		
3	97.	90.224		
4	48.	45.112		
5	16.	18.045		
6 to INF	9.	8.282		
Chisquare with	6 d.o.f.	= 5.18	p-value=	.479490
	For a s	ample of size	ze 500:	mean
bloc	k0.rng	using bits	8 to 31	
duplicate	number	number expected		
spacings	observed	expected	b	
0	68.			
1	151.			
2	125.	135.335		
3	81.	90.224		
4	49.	45.112		
5	15.	18.045		
6 to INF	11.	8.282		
Chisquare with	6 d.o.f.	= 5.29	o-value=	.492559
	: : : : : : : : : : : :		:::::	
	For a s	ample of size	ze 500:	mean
bloc	k0.rng	ample of sis using bits	9 to 32	2.026
duplicate	number	number		
		expected	b	
0	63.	67.668		
1	141.	135.335		
2	133.	135.335		
3	82.	90.224		
4	53.	45.112		
5	21.	18.045		
6 to INF	7.	8.282		
6 to INF Chisquare with	6 d.o.f.	= 3.41	o-value=	.244183
	: : : : : : : : : : : :		: : : : : :	
The 9 p-valu	les were			
		.116675	.937734	.049389
.815725	.479490	.116675 .492559	.244183	
A KSTEST for	the 9 p-val	ues yields	.079935	

\$

:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test ::

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:: equivalent to the likelihood ratio test that the 120 cell
                                                         ::
    :: counts came from the specified (asymptotically) normal dis- ::
    :: tribution with the specified 120x120 covariance matrix (with ::
    :: rank 99). This version uses 1,000,000 integers, twice.
                                                         ::
    OPERM5 test for file block0.rng
    For a sample of 1,000,000 consecutive 5-tuples,
chisquare for 99 degrees of freedom= 71.452; p-value= .016727
        OPERM5 test for file block0.rng
    For a sample of 1,000,000 consecutive 5-tuples,
chisquare for 99 degrees of freedom=107.158; p-value= .729698
    :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost ::
    :: 31 bits of 31 random integers from the test sequence are used ::
    :: to form a 31x31 binary matrix over the field \{0,1\}. The rank ::
    :: is determined. That rank can be from 0 to 31, but ranks< 28
                                                         ::
    :: are rare, and their counts are pooled with those for rank 28. ::
    :: Ranks are found for 40,000 such random matrices and a chisqua-::
    :: re test is performed on counts for ranks 31,30,29 and <=28. ::
    Binary rank test for block0.rng
       Rank test for 31x31 binary matrices:
      rows from leftmost 31 bits of each 32-bit integer
          observed expected (o-e)^2/e sum
    rank
                    211.4 1.633211
      28
             230
                                   1.633
      29
             5098
                   5134.0
                         .252578
                                    1.886
                          .099922
            23055
                  23103.0
      30
                                    1.986
                  11551.5 .371124
      31
            11617
                                    2.357
 chisquare= 2.357 for 3 d. of f.; p-value= .560648
     _____
    :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x ::
    :: 32 binary matrix is formed, each row a 32-bit random integer. ::
    :: The rank is determined. That rank can be from 0 to 32, ranks ::
    :: less than 29 are rare, and their counts are pooled with those ::
    :: for rank 29. Ranks are found for 40,000 such random matrices ::
    :: and a chisquare test is performed on counts for ranks 32,31, ::
    :: 30 and <=29.
                                                         ::
    Binary rank test for block0.rng
       Rank test for 32x32 binary matrices:
      rows from leftmost 32 bits of each 32-bit integer
    rank observed expected (o-e)^2/e sum
      29
             179
                    211.4 4.970852 4.971
      30
             5046
                   5134.0 1.508724
                                  6.480
      31
            23244
                  23103.0 .859964
                                   7.340
      32
            11531
                  11551.5 .036467
                                    7.376
 chisquare= 7.376 for 3 d. of f.; p-value= .942474
:: This is the BINARY RANK TEST for 6x8 matrices. From each of ::
    :: six random 32-bit integers from the generator under test, a
                                                         ::
    :: specified byte is chosen, and the resulting six bytes form a ::
```

:: 6x8 binary matrix whose rank is determined. That rank can be ::

:: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: random matrices, and a chi-square test is performed on :: :: counts for ranks 6,5 and <=4. :: Binary Rank Test for block0.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 983 944.3 1.586 1.586 r =5 21743 21743.9 .000 1.586 r =6 77274 77311.8 .018 1.604 p=1-exp(-SUM/2)=.55167Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .008 947 944.3 .008 r<=4 21737 21743.9 .002 .010 r =5 r =6 77316 77311.8 .000 .010 p=1-exp(-SUM/2)=.00505Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 937 944.3 .056 .056 r =5 21629 21743.9 .607 .664 r =б 77434 77311.8 .193 .857 p=1-exp(-SUM/2)=.34843Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 4 to 11 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 939 944.3 .030 .030 .034 r =5 21771 21743.9 .064 77290 .006 .070 r =6 77311.8 p=1-exp(-SUM/2)=.03424Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .008 947 944.3 .008 r<=4 21644 21743.9 .459 .467 r =5 77409 77311.8 .122 .589 r =6 p=1-exp(-SUM/2)=.25505Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 935 944.3 .092 .092 r =5 21715 21743.9 .038 .130 r =6 77350 77311.8 .019 .149 p=1-exp(-SUM/2)=.07175Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 7 to 14

OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 943 944.3 .002 .002 21622 21743.9 r =5 .683 .685 r =б 77435 77311.8 .196 .881 p=1-exp(-SUM/2)=.35645Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 8 to 15 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 954 944.3 .100 .100 21723 .020 .120 r =5 21743.9 r =6 77323 77311.8 .002 .121 p=1-exp(-SUM/2)=.05886Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .001 .001 r<=4 945 944.3 21999 21743.9 2.993 2.993 r =5 77056 77311.8 .846 3.840 r =6 p=1-exp(-SUM/2)=.85337Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 950 944.3 .034 .034 21743.9 r =5 21714 .041 .076 r =6 77336 77311.8 .008 .083 p=1-exp(-SUM/2)=.04069Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 940 944.3 .020 .020 21783 21743.9 .070 .090 r =5 .016 r =6 77277 77311.8 .106 p=1-exp(-SUM/2)=.05142Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .546 r<=4 967 944.3 .546 .767 r =5 21873 21743.9 1.312 77160 77311.8 .298 1.610 r =6 p=1-exp(-SUM/2)=.55296Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 956 944.3 .145 r<=4 .145 21747 21743.9 r =5 .000 .145 r =б 77297 77311.8 .003 .148 p=1-exp(-SUM/2)=.07143Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 14 to 21 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM

944.3 r<=4 932 .160 .160 21743.9 r =5 21683 .171 .331 .400 r =6 77385 77311.8 .069 p=1-exp(-SUM/2)=.18132Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .355 r<=4 926 944.3 .355 r =5 21735 21743.9 .004 .358 77339 r =6 77311.8 .010 .368 p=1-exp(-SUM/2)=.16802Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 16 to 23 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .972 r<=4 914 944.3 .972 .029 r =5 21719 21743.9 1.001 .039 77367 77311.8 1.040 r =6 p=1-exp(-SUM/2)=.40555Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 943 .002 .002 r<=4 21743.9 r =5 21508 2.559 2.561 r =6 77549 77311.8 .728 3.289 p=1-exp(-SUM/2)=.80687Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 18 to 25 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 929 944.3 .248 .248 r =5 21505 21743.9 2.625 2.873 77566 r =б 77311.8 .836 3.709 p=1-exp(-SUM/2)=.84343Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 19 to 26 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .006 r<=4 942 944.3 .006 r =5 21726 21743.9 .015 .020 77332 r =6 77311.8 .005 .026 p=1-exp(-SUM/2)=.01273Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 20 to 27  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM r<=4 924 944.3 .436 .436 r =5 21811 21743.9 .207 .644 r =6 77265 77311.8 .028 .672 p=1-exp(-SUM/2)=.28532Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 953 944.3 .080 .080

21648 21743.9 .503 r =5 .423 r =б 77399 77311.8 .098 .601 p=1-exp(-SUM/2)=.25971Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 904 944.3 1.720 1.720 r<=4 r =5 21816 21743.9 .239 1.959 r =б 77280 77311.8 .013 1.972 p=1-exp(-SUM/2)=.62696Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 921 944.3 .575 .575 21868 21743.9 r =5 .708 1.283 .131 r =6 77211 77311.8 1.415 p=1-exp(-SUM/2)=.50705Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 969 944.3 .646 .646 r =5 21743.9 3.764 4.410 22030 77001 77311.8 1.249 5.660 r =б p=1-exp(-SUM/2)=.94098Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block0.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .030 r<=4 939 944.3 .030 21846 21743.9 .479 .509 r =5 r =6 77215 77311.8 .121 .630 p=1-exp(-SUM/2)=.27035TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .255055 .551670 .005052 .348434 .034243 .071746 .356446 .058859 .853374 .040689 .051416 .552956 .071426 .181316 .168023 .405552 .806872 .843431 .012728 .285323 .259714 .626962 .507048 .940984 .270353 brank test summary for block0.rng The KS test for those 25 supposed UNI's yields KS p-value= .995340 :: THE BITSTREAM TEST :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... . Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter ::

:: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 ::

::

:: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: (j-141909)/428 should be a standard normal variate (z score) :: :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. : : THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^{21}$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ tst no 1: 141905 missing words, -.01 sigmas from mean, p-value= .49597 tst no 2: 141555 missing words, -.83 sigmas from mean, p-value= .20387 tst no 3: 142103 missing words, .45 sigmas from mean, p-value= .67455 tst no 4: 141847 missing words, -.15 sigmas from mean, p-value= .44211 tst no 5: 142169 missing words, .61 sigmas from mean, p-value= .72798 140896 missing words, tst no 6: -2.37 sigmas from mean, p-value= .00895

tst no 7: 142178 missing words, .63 sigmas from mean, p-value= .73491 tst no 8: 142889 missing words, 2.29 sigmas from mean, p-value= .98896 tst no 9: 141382 missing words, -1.23 sigmas from mean, p-value= .10896 tst no 10: 141974 missing words, .15 sigmas from mean, p-value= .56005 tst no 11: 142468 missing words, 1.31 sigmas from mean, p-value= .90411 141087 missing words, tst no 12: -1.92 sigmas from mean, p-value= .02735 tst no 13: 142862 missing words, 2.23 sigmas from mean, p-value= .98699 141593 missing words, tst no 14: -.74 sigmas from mean, p-value= .22993 tst no 15: 142527 missing words, 1.44 sigmas from mean, p-value= .92551 tst no 16: 141849 missing words, -.14 sigmas from mean, p-value= .44395 tst no 17: 141384 missing words, -1.23 sigmas from mean, p-value= .10984 tst no 18: 142783 missing words, 2.04 sigmas from mean, p-value= .97939 -.73 sigmas from mean, p-value= .23421 tst no 19: 141599 missing words, tst no 20: 141377 missing words, -1.24 sigmas from mean, p-value= .10679

:: The tests OPSO, OQSO and DNA :: :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: :: file, elements of which are assumed 32-bit random integers. :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2<sup>21+3</sup> "keystrokes"), is again 141909, with ::

:: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: integers being tested. It considers 10-letter words, so that :: :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block0.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw Ζ р .8758 OPSO for block0.rng using bits 23 to 32 142244 1.154 .433 .6676 OPSO for block0.rng using bits 22 to 31 142035 OPSO for block0.rng using bits 21 to 30 141494 -1.432 .0760 OPSO for block0.rng using bits 20 to 29 141726 -.632 .2636 141593 -1.091 OPSO for block0.rng using bits 19 to 28 .1377 .502 OPSO for block0.rng using bits 18 to 27 142055 .6923 OPSO for block0.rng using bits 17 to 26 141315 -2.049 .0202 OPSO for block0.rng using bits 16 to 25 141424 -1.674 .0471 using bits 15 to 24 OPSO for block0.rng 141745 -.567 .2855 using bits 14 to 23 OPSO for block0.rng -.267 141832 .3949 using bits 13 to 22 OPSO for block0.rng 141643 -.918 .1792 OPSO for block0.rng using bits 12 to 21 141954 .154 .5612 .261 OPSO for block0.rng using bits 11 to 20 141985 .6029 -.115 OPSO for block0.rng using bits 10 to 19 141876 .4543 .8938 OPSO for block0.rng using bits 9 to 18 142271 1.247 .309 OPSO for block0.rng using bits 8 to 17 141999 .6214 OPSO for block0.rng using bits 7 to 16 141869 -.139 .4447 OPSO for block0.rng using bits 6 to 15 141857 -.180 .4284 OPSO for block0.rng using bits 5 to 14 141856 -.184 .4270 OPSO for block0.rng .764 using bits 142131 4 to 13 .7777 OPSO for block0.rng using bits 142168 3 to 12 .892 .8138 OPSO for block0.rng using bits .7219 2 to 11 142080 .589 OPSO for block0.rng using bits 1 to 10 141612 -1.025 .1526 OQSO test for generator block0.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw 7. р -.805 OQSO for block0.rng using bits 28 to 32 141672 .2106 OQSO for block0.rng using bits 27 to 31 141611 -1.011 .1559 OQSO for block0.rng using bits 26 to 30 142281 1.260 .8961 OQSO for block0.rng using bits 25 to 29 141774 -.459 .3232 OQSO for block0.rng using bits 24 to 28 142034 .423 .6637 OQSO for block0.rng using bits 23 to 27 142468 1.894 .9709 using bits 22 to 26 OQSO for block0.rng 142100 .7410 .646 OQSO for block0.rng using bits 21 to 25 141932 .077 .5306 OQSO for block0.rng using bits 20 to 24 142181 .921 .8215 OQSO for block0.rng using bits 19 to 23 141312 -2.025 .0214 OQSO for block0.rng using bits 18 to 22 141852 -.194 .4230 OQSO for block0.rng using bits 17 to 21 142487 1.958 .9749 .558 OQSO for block0.rng using bits 16 to 20 142074 .7116 using bits 15 to 19 142282 1.263 OQSO for block0.rng .8968 OQSO for block0.rng using bits 14 to 18 142175 .901 .8161

OQSO	for	block0.rng	using	bits	13	to	17	141888	072	.4712
OQSO	for	block0.rng	using	bits	12	to	16	141469	-1.493	.0678
OQSO	for	block0.rng	using	bits	11	to	15	142293	1.301	.9033
OQSO	for	block0.rng	using	bits	10	to	14	142003	.318	.6246
OQSO	for	block0.rng	using	bits	9	to	13	141766	486	.3135
OQSO	for	block0.rng	using	bits	8	to	12	141867	143	.4430
OQSO	for	block0.rng	using	bits	7	to	11	142185	.934	.8250
OQSO	for	block0.rng	using	bits	6	to	10	142017	.365	.6424
OQSO	for	block0.rng	using	bits	5	to	9	142440	1.799	.9640
OQSO	for	block0.rng	using	bits	4	to	8	141971	.209	.5828
		block0.rng	using	bits	3	to	7	141716	655	.2561
OQSO	for	block0.rng	using	bits	2	to	6	141133	-2.632	.0042
OQSO	for	block0.rng	using	bits	1	to	5	142095	.629	.7355
		or generator blo	-							
		. missing words			noi	rmal	l variate	(z), p-	value (p	)
-		5		-				mw	Z	р
DNA	for	block0.rng	using	bits	31	to	32	142113	.601	.7260
DNA	for	block0.rng	using	bits	30	to	31	142436	1.554	.9399
DNA	for	block0.rng	using	bits	29	to	30	142500	1.742	.9593
DNA	for	block0.rng	using	bits	28	to	29	141793	343	.3657
DNA	for	block0.rng	using	bits	27	to	28	142059	.442	.6706
		block0.rng	using					141946	.108	.5431
DNA	for	block0.rng	using	bits	25	to	26	142110	.592	.7231
DNA	for	block0.rng	using					142596	2.026	.9786
		block0.rng	using					142252	1.011	.8440
DNA	for	block0.rng	using	bits	22	to	23	142398	1.442	.9253
DNA	for	block0.rng	using	bits	21	to	22	141887	066	.4737
DNA	for	block0.rng	using	bits	20	to	21	141697	626	.2655
DNA	for	block0.rng	using	bits	19	to	20	141989	.235	.5929
DNA	for	block0.rng	using	bits	18	to	19	141340	-1.679	.0465
DNA	for	block0.rng	using	bits	17	to	18	142239	.972	.8346
DNA	for	block0.rng	using	bits	16	to	17	141424	-1.432	.0761
DNA	for	block0.rng	using	bits	15	to	16	142643	2.164	.9848
DNA	for	block0.rng	using	bits	14	to	15	141879	089	.4644
DNA	for	block0.rng	using	bits	13	to	14	142159	.736	.7693
DNA	for	block0.rng	using	bits	12	to	13	141770	411	.3405
DNA	for	block0.rng	using	bits	11	to	12	141704	606	.2724
DNA	for	block0.rng	using	bits	10	to	11	141889	060	.4761
DNA	for	block0.rng	using			to		142157	.731	.7675
DNA	for	block0.rng	using	bits	8	to	9	141965	.164	.5652
DNA	for	block0.rng	using	bits	7	to	8	141840	205	.4190
		block0.rng	using		б	to	7	142055	.430	.6663
		block0.rng	using			to	6	142161	.742	.7711
		block0.rng	using			to	5	142031	.359	.6402
		block0.rng	using			to	4	142048	.409	.6588
		block0.rng	using			to	3		-1.685	.0460
		block0.rng	using			to	2	141970	.179	.5710
		5	5			-			-	-

#### 

:: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5<sup>5</sup> :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: :: :: for each word. The quadratic form in the weak inverse of :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block0.rng Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block0.rng 2446.69 -.754 .225440 byte stream for block0.rng 2534.80 .492 .688677 :: This is the COUNT-THE-1's TEST for specific bytes. :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left-:: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: .251 bits 1 to 8 2517.72 .598921 bits 2 to 9 2482.71 -.245 .403390 bits 3 to 10 2576.00 1.075 .858760 bits 4 to 11 2488.60 -.161 .435936 bits 5 to 12 2513.93 .197 .578093 bits 6 to 13 2552.13 .737 .769492 bits 7 to 14 2562.49 .884 .811579 bits 8 to 15 2508.38 .119 .547195 bits 9 to 16 2481.69 -.259 .397849

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-1.161 .727

-.419

.354

-.137

.122852

.766406

.337613

.638356

.445373

bits 10 to 17

bits 11 to 18

bits 12 to 19

bits 13 to 20

bits 14 to 21

2417.92

2551.41

2470.37

2525.04

2490.29

bits 1	5 to	22	2564.41	.911	.818810
bits 10	5 to	23	2557.52	.813	.792012
bits 1	7 to	24	2518.44	.261	.602889
bits 18	3 to	25	2373.56	-1.788	.036877
bits 19	) to	26	2483.70	230	.408872
bits 20	) to	27	2532.05	.453	.674809
bits 21	L to	28	2480.42	277	.390920
bits 22	2 to	29	2496.15	055	.478261
bits 23	3 to	30	2489.79	144	.442617
bits 24	l to	31	2460.15	564	.286518
bits 25	5 to	32	2599.73	1.410	.920780

:: THIS IS A PARKING LOT TEST :: :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect ::  $\colon\colon$  random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st-:: :: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block0.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 Successes: 3532 z-score: .411 p-value: .659449 Successes: 3489 z-score: -1.553 p-value: .060270 Successes: 3504 z-score: -.868 p-value: .192812 Successes: 3523 z-score: .000 p-value: .500000 Successes: 3517 z-score: -.274 p-value: .392053 Successes: 3528 z-score: .228 p-value: .590298 Successes: 3521 z-score: -.091 p-value: .463618 Successes: 3508 -.685 p-value: .246694 z-score: Successes: 3517 z-score: -.274 p-value: .392053 Successes: 3565 z-score: 1.918 p-value: .972432 square size avg. no. parked sample sigma 100. 3520.400 19.012 KSTEST for the above 10: p= .300390

### \$

<pre>:: square :: the (n^ :: pendent :: should 1 :: .995 :: a KSTES :: formity :: are prime: are prime: random</pre>	this 100 of side 1 2-n)/2 pa uniform, be (very Thus 1-e I on the for rand nted but choices c	times:: 0000. Fir irs of point then d^2, close to) exp(-d^2/.9 resulting om points the KSTEST of 8000 point	choose n=8000 random points in a : nd d, the minimum distance between : ints. If the points are truly inde- ; the square of the minimum distance : exponentially distributed with mean : 995) should be uniform on [0,1) and : 100 values serves as a test of uni- in the square. Test numbers=0 mod 5 : I is based on the full set of 100 :	:::::::::::::::::::::::::::::::::::::::
	This is t	he MINIMUN	M DISTANCE test	
f	or random	integers	in the file block0.rng	
Sample no.	d^2	avg	equiv uni	
5	.1127	.6202	.107114	
10	.3265	.5722	.279752	
15	1.5969	.8933	.799101	
20	.4793	.8284	.382297	
25	.0679	.7958	.065992	
30	3.1047	.9657	.955856	
35	1.0304	1.0735	.644983	
40	.3540	1.0087	.299374	
45	1.3808	.9832	.750369	
50	.9728	.9870	.623819	
55	4.6340	1.0585	.990508	
60	.8098	1.0668	.556858	
65	1.0949	1.0389	.667256	
70	.3421	.9843	.290915	
75	.0461	.9883	.045309	
80	1.3411	.9657	.740190	
85	.3548	.9493	.299955	
90	.5121	1.0054	.402281	
95	.5599	.9958	.430317	
100	.7538	.9921	.531210	

MINIMUM DISTANCE TEST for block0.rng Result of KS test on 20 transformed mindist^2's: p-value= .223584

#### 

:: THE 3DSPHERES TEST :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of 1-exp(-r^3/30.), then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block0.rng r^3= 23.673 p-value= .54575 sample no: 1 sample no: 2 r^3= 5.165 p-value= .15816 r^3= 38.008 sample no: 3 p-value= .71830

```
sample no: 4
                r^3= 29.919
                                 p-value= .63112
                               p-value= .48142
p-value= .85362
             r<sup>3</sup>= 19.700
r<sup>3</sup>= 57.646
                r^3= 19.700
sample no: 5
sample no: 6
             r^3= 57.646 p-value= .85362
r^3= 14.649 p-value= .38632
r^3= 84.430 p-value= .94006
r^3= 10.395 p-value= .29283
r^3= 15.515 p-value= .40378
sample no: 7
sample no: 8
sample no: 9
sample no: 10
sample no: 11
               r^3= 20.948 p-value= .50255
sample no: 12
               r^3= 81.798
                               p-value= .93456
               r^3= 16.334
sample no: 13
                               p-value= .41984
sample no: 14
               r^3= 31.497
                                p-value= .65003
               r<sup>3</sup>= 3.279
r<sup>3</sup>= 7.895
sample no: 15
                                p-value= .10354
sample no: 16
                                p-value= .23140
                                p-value= .71299
sample no: 17
                r^3= 37.448
             r<sup>3</sup>= 61.286 p-value= .87034
r<sup>3</sup>= 5.064 p-value= .15533
r<sup>3</sup>= 14.611 p-value= .38555
sample no: 18
                                p-value= .87034
sample no: 19
sample no: 20
 A KS test is applied to those 20 p-values.
_____
      3DSPHERES test for file block0.rng p-value= .133599
::
           This is the SQEEZE test
    :: Random integers are floated to get uniforms on [0,1). Start- ::
    :: ing with k=2^{31}=2147483647, the test finds j, the number of ::
    :: iterations necessary to reduce k to 1, using the reduction
                                                                 ::
    ::
       k=ceiling(k*U), with U provided by floating integers from
                                                                 ::
    :: the file being tested. Such j's are found 100,000 times,
                                                                 ::
    :: then counts for the number of times j was <=6,7,\ldots,47,>=48 ::
    :: are used to provide a chi-square test for cell frequencies. ::
    RESULTS OF SQUEEZE TEST FOR block0.rng
        Table of standardized frequency counts
    ( (obs-exp)/sqrt(exp) )^2
      for j taking values <=6,7,8,...,47,>=48:
           1.8 -.4 -1.1 1.1
                                         .2
    -.1
                  .6
.6
     .2
            .8
                                         1.1
    1.1
           1.0
                                         -.8
    -.2
           -.7
                                         -.б
                                         .6
    -.3
           -.2
    1.2
           -.5
                                         -1.7
                -1.2 -1.8
     .3
            .8
                                         1.0
    -.1
          Chi-square with 42 degrees of freedom: 29.384
             z-score= -1.377 p-value= .070700
```

#### 

:: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: Test no. 1 p-value .024581 Test no. 2 p-value .783819 Test no. 3 p-value .126377 Test no. 4 p-value .805804 Test no. 5 p-value .956854 Test no. 6 p-value .747899 Test no. 7 p-value .738569 p-value .332539 Test no. 8 Test no. 9 p-value .290589 Test no. 10 p-value .241737 Results of the OSUM test for block0.rng KSTEST on the above 10 p-values: .162389 

This is the RUNS test. It counts runs up, and runs down, :: :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: :: repeated. The RUNS test for file block0.rng Up and down runs in a sample of 10000

:

Run test for block0.rng runs up; ks test for 10 p's: .895588 runs down; ks test for 10 p's: .103022 Run test for block0.rng runs up; ks test for 10 p's: .689373 runs down; ks test for 10 p's: .526976

#### 

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with :: :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for block0.rng

No. of wins:	Observ	ved Expect	ed			
			98204	98585.86		
	9820	4= No. of	wins, z-	score=-1.	708 pvalue=	.04383
Analysis of	Throws-	per-Game:				
Chisq= 23.90	for 20	degrees c	of freedom	, p= .75	316	
	Throws	Observed	Expected	Chisq	Sum	
	1	66462	66666.7	.628	.628	
	2	37635	37654.3	.010	.638	
	3	26885	26954.7	.180	.819	
	4	19335	19313.5	.024	.843	
	5	13990	13851.4	1.386	2.229	
	б	10053	9943.5	1.205	3.434	
	7	7095	7145.0	.350	3.784	
	8		5139.1		3.784	
	9	3733	3699.9	.297	4.081	
	10		2666.3			
	11	1904	1923.3	.194	8.232	
	12	1364	1388.7	.441	8.672	
	13	1038	1003.7	1.171	9.843	
	14	715	726.1	.171	10.014	
	15	508	525.8	.605	10.619	
	16	404	381.2	1.370	11.989	
	17	258	276.5			
	18	163	200.8	7.126	20.358	
	19	156	146.0	.687	21.045	
	20	121		2.058		
	21	272	287.1	.796	23.899	
SUI		OR block0	5			
	p-valu	le for no.	of wins:	.043828		
		o for thr	courd / domo .	752150		

p-value for throws/game: .753159

#### 

Results of DIEHARD battery of tests sent to file report0.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p> .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". This is the BIRTHDAY SPACINGS TEST :: :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically ::

:: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block1.rng For a sample of size 500: mean using bits block1.rng 1 to 24 2.016 duplicate number number spacings observed expected 67.668 56. 0 1 145. 135.335 2 141. 135.335 3 90.224 93. 4 40. 45.112 5 12. 18.045 6 to INF 13. 8.282 Chisquare with 6 d.o.f. = 8.32 p-value= .784188 For a sample of size 500: mean block1.rng using bits 2 to 25 2.038 duplicate number number spacings observed expected 0 66. 67.668 1 132. 135.335 2 140. 135.335 3 83. 90.224 4 45.112 51. 5 18.045 18. 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =1.99 p-value= .079160 For a sample of size 500: mean block1.rng using bits 3 to 26 1.930 duplicate number number spacings observed expected 0 73. 67.668 1 148. 135.335 2 145. 135.335 3 90.224 61. 4 45.112 42. 5 18.045 18. 6 to INF 13. 8.282 .976961 Chisquare with 6 d.o.f. = 14.66 p-value= For a sample of size 500: mean 1.912 using bits 4 to 27 block1.rng duplicate number number

observed spacings expected 67.668 0 81. 135.335 1 132. 2 141. 135.335 3 76. 90.224 4 48. 45.112 5 14. 18.045 6 to INF 8. 8.282 Chisquare with 6 d.o.f. = 6.29 p-value= .608453 For a sample of size 500: mean block1.rng using bits 5 to 28 1.996 duplicate number number spacings observed expected 0 63. 67.668 1 136. 135.335 2 141. 135.335 3 88. 90.224 4 52. 45.112 18.045 5 13. 6 to INF 8.282 7. Chisquare with 6 d.o.f. =3.28 p-value= .226722 For a sample of size 500: mean block1.rng using bits 6 to 29 2.016 duplicate number number expected spacings observed 0 60. 67.668 1 133. 135.335 2 148. 135.335 3 89. 90.224 4 45. 45.112 5 19. 18.045 6 to INF 6. 8.282 Chisquare with 6 d.o.f. =2.79 p-value= .165348 For a sample of size 500: mean block1.rng using bits 7 to 30 1.898 number duplicate number spacings observed expected 88. 67.668 0 1 127. 135.335 2 122. 135.335 3 92. 90.224 4 56. 45.112 5 13. 18.045 8.282 6 to INF 2. Chisquare with 6 d.o.f. = 16.77 p-value= .989853 For a sample of size 500: mean block1.rng using bits 8 to 31 2.052 duplicate number number spacings observed expected 0 66. 67.668 1 126. 135.335 2 145. 135.335 3 90.224 91.

4 37. 45.112 5 18.045 23. 6 to INF 12. 8.282 Chisquare with 6 d.o.f. =5.87 p-value= .562162 For a sample of size 500: mean using bits 9 to 32 1.940 block1.rng duplicate number number spacings observed expected 0 73. 67.668 1 141. 135.335 2 130. 135.335 3 96. 90.224 4 34. 45.112 5 14. 18.045 6 to INF 12. 8.282 Chisquare with 6 d.o.f. = 6.55 p-value= .635570 The 9 p-values were .784188 .079160 .976961 .608453 .226722 .165348 .989853 .562162 .635570 A KSTEST for the 9 p-values yields .514385

\$

:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block1.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=111.433; p-value= .814967 OPERM5 test for file block1.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=102.642; p-value= .619097 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block1.rng Rank test for 31x31 binary matrices:

rows from leftmost 31 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 211.4 .055259 5134.0 .467470 .055 28 208 .467470 .523 29 5183 .712 23037 23103.0 .188814 11572 11551.5 .036294 30 .748 31 chisquare= .748 for 3 d. of f.; p-value= .331683 \_\_\_\_\_ :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. :: Binary rank test for block1.rng Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 29 216 211.4 .099304 .099 5134.0 1.721447 1.821 30 5040 23289 23103.0 1.496710 31 3.317 11455 11551.5 .806557 4.124 32 chisquare= 4.124 for 3 d. of f.; p-value= .772759 \_\_\_\_\_ :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: random matrices, and a chi-square test is performed on :: :: counts for ranks 6,5 and <=4. :: Binary Rank Test for block1.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED (O-E)^2/E SUM .023 .023 r<=4 949 944.3 21743.9 r =5 21640 .496 .520 77411 77311.8 .127 r =6 .647 p=1-exp(-SUM/2)=.27643Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 2 to 9 SUM OBSERVED EXPECTED  $(O-E)^{2}/E$ 944.3 961 .295 .295 r<=4 2154621743.91.8017749377311.8.425 r =5 2.096 r =6 2.521 p=1-exp(-SUM/2)=.71651

Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .080 .080 r<=4 953 944.3 r =5 21567 21743.9 1.439 1.519 77480 77311.8 1.885 r =6 .366 p=1-exp(-SUM/2)=.61040Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 962 944.3 .332 .332 r =5 21532 21743.9 2.065 2.397 r =6 77506 77311.8 .488 2.885 p=1-exp(-SUM/2)=.76361Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 950 944.3 .034 .034 .294 r =5 21664 21743.9 .328 r =6 77386 77311.8 .071 .399 p=1-exp(-SUM/2)=.18094Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 937 944.3 .056 .056 .331 r =5 21659 21743.9 .388 77404 77311.8 .110 .498 r =6 p=1-exp(-SUM/2)=.22038Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 7 to 14  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM 1.986 1.986 901 944.3 r<=4 r =5 21810 21743.9 .201 2.187 r =6 77289 77311.8 .007 2.193 p=1-exp(-SUM/2)=.66601Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 932 944.3 .160 .160 r =5 21701 21743.9 .085 .245 r =6 77367 77311.8 .039 .284 p=1-exp(-SUM/2)=.13251Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 9 to 16 OBSERVED  $(O-E)^{2}/E$ EXPECTED SUM r<=4 978 944.3 1.203 1.203 .034 r =5 21771 21743.9 1.236 77311.8 .048 r =6 77251 1.284 p=1-exp(-SUM/2)=.47381Rank of a 6x8 binary matrix,

rows formed from eight bits of the RNG block1.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 7.068 r<=4 1026 944.3 7.068 21743.9 r =5 21771 .034 7.102 .153 77203 77311.8 7.255 r =6 p=1-exp(-SUM/2)=.97342Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 11 to 18 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 959 944.3 .229 . 229 r =5 21803 21743.9 .161 .389 77238 77311.8 .070 .460 r =б p=1-exp(-SUM/2)=.20542Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 986 1.841 r<=4 944.3 1.841 r =5 21643 21743.9 .468 2.310 77371 77311.8 .045 2.355 r =6 p=1-exp(-SUM/2)=.69193Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 991 944.3 2.309 2.309 r =5 21666 21743.9 .279 2.588 .013 r =6 77343 77311.8 2.601 p=1-exp(-SUM/2)=.72762Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 14 to 21 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 965 944.3 .454 .454 r<=4 r =5 21684 21743.9 .165 .619 77351 77311.8 r =б .020 .639 p=1-exp(-SUM/2)=.27334Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 919 944.3 .678 .678 r<=4 r =5 21708 21743.9 .059 .737 r =6 77373 77311.8 .048 .786 p=1-exp(-SUM/2)=.32484Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 16 to 23 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r <= 4900 944.3 2.078 2.078 21586 21743.9 r =5 1.147 3.225 3.754 77514 77311.8 .529 r =6 p=1-exp(-SUM/2)=.84694Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng

b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 914 944.3 .972 .972 r =5 21643 21743.9 .468 1.441 77443 77311.8 .223 r =6 1.663 p=1-exp(-SUM/2)=.56464Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 18 to 25 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 989 944.3 2.116 2.116 r =5 21855 21743.9 .568 2.683 r =6 77156 77311.8 .314 2.997 p=1-exp(-SUM/2)=.77659Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 19 to 26 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 1032 944.3 8.145 8.145 r<=4 21790 21743.9 .098 8.242 r =5 r =6 77178 77311.8 .232 8.474 p=1-exp(-SUM/2)=.98555Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 20 to 27 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 939 944.3 .030 .030 r =5 21558 21743.9 1.589 1.619 r =б 77503 77311.8 .473 2.092 p=1-exp(-SUM/2)=.64865Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 964 944.3 .411 .411 .741 r =5 21617 21743.9 1.152 77419 77311.8 .149 1.300 r =6 p=1-exp(-SUM/2)=.47800Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .008 .008 r<=4 947 944.3 21627 21743.9 .628 .636 r =5 77426 r =6 77311.8 .169 .805

p=1-exp(-SUM/2)=.33131Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 939 944.3 .030 .030 r =5 21854 21743.9 .557 .587 .142 r =6 77207 77311.8 .729 p=1-exp(-SUM/2)=.30557Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng

b-rank test for bits 24 to 31

OBSERVED EXPECTED (O-E)^2/E SUM 944.3 r<=4 1010 4.571 4.571 21743.9 .253 r =5 21818 4.823 r =6 77172 77311.8 .253 5.076 p=1-exp(-SUM/2)=.92099Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block1.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED (O-E)^2/E SUM .436 r<=4 924 944.3 .436 21847 21743.9 .489 r =5 .925 r =6 77229 77311.8 .089 1.014 p=1-exp(-SUM/2)=.39770TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .716507 .610395 .763609 .180941 .276434 .666005 .132506 .473809 .727615 .273340 .220379 .973421 .691935 .324845 .205421 .985549 .564643 .776588 .648652 .846937 .331311 .305568 .397697 .477998 .920986 brank test summary for block1.rng The KS test for those 25 supposed UNI's yields KS p-value= .414012

## 

:: THE BITSTREAM TEST :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... . Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^21$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ tst no 1: 142400 missing words, 1.15 sigmas from mean, p-value= .87419 tst no1:142400 missing words,<br/>tst no1.15 sigmas from mean, p-value=.87419tst no2:141534 missing words,<br/>tst no-.88 sigmas from mean, p-value=.19026tst no3:142040 missing words,<br/>tst no-.88 sigmas from mean, p-value=.19026tst no4:141799 missing words,<br/>tst no-.26 sigmas from mean, p-value=.39829tst no5:142154 missing words,<br/>tst no.57 sigmas from mean, p-value=.71622tst no6:141972 missing words,<br/>tst no.57 sigmas from mean, p-value=.5821tst no7:141667 missing words,<br/>tst no.57 sigmas from mean, p-value=.28563tst no8:141822 missing words,<br/>tst no.20 sigmas from mean, p-value=.41916tst no10:142531 missing words,<br/>tst no1.45 sigmas from mean, p-value=.92682.33 sigmas from mean, p-value=.63056

tst no	12:	141780	missing	words,	30	sigmas	from meas	n, p-value=	.38126
tst no	13:	141425	missing	words,	-1.13	sigmas	from meas	n, p-value=	.12890
tst no	14:	141711	missing	words,	46	sigmas	from mea	n, p-value=	.32154
tst no	15:	141938	missing	words,	.07	sigmas	from meas	n, p-value=	.52671
tst no	16:	141950	missing	words,	.10	sigmas	from meas	n, p-value=	.53785
tst no	17:	141412	missing	words,	-1.16	sigmas	from mea	n, p-value=	.12262
tst no	18:	141698	missing	words,	49	sigmas	from mea	n, p-value=	.31074
tst no	19:	142031	missing	words,	.28	sigmas	from meas	n, p-value=	.61190
tst no	20:	141678	missing	words,	54	sigmas	from meas	n, p-value=	.29443

:: The tests OPSO, OQSO and DNA :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: :: extensive simulation. :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: integers being tested. It considers 10-letter words, so that :: :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. : : OPSO test for generator block1.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р OPSO for block1.rng using bits 23 to 32 141950 .140 .5558 OPSO for block1.rng using bits 22 to 31 141581 -1.132 .1288 OPSO for block1.rng using bits 21 to 30 141494 -1.432 .0760 OPSO for block1.rng using bits 20 to 29 142281 1.282 .9000 OPSO for block1.rng using bits 19 to 28 141780 -.446 .3278

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using bits 18 to 27

142163

.875

.8091

OPSO for block1.rng

OPSO for	block1.rng	using	bits	17	to	26	142118	.720	.7641
OPSO for	block1.rng	using	bits	16	to	25	141461	-1.546	.0611
OPSO for	block1.rng	using	bits	15	to	24	141481	-1.477	.0698
OPSO for	block1.rng	using	bits	14	to	23	142264	1.223	.8893
OPSO for	block1.rng	using	bits	13	to	22	141703	711	.2384
	block1.rng	using					141614	-1.018	.1543
OPSO for	block1.rng	using	bits	11	to	20	142151	.833	.7977
	block1.rng	using					141895	049	.4803
	block1.rng	using			to		141928		.5257
	block1.rng	using			to		142159		.8054
	block1.rng	using			to		142062	.526	.7007
	block1.rng	using			to			-1.436	.0756
	block1.rng	using			to			-1.294	.0978
	block1.rng	using			to		142510		.9808
	block1.rng	using			to		141904		.4927
	block1.rng	using			to		141845		.4122
	block1.rng	using			to			805	
	or generator blog			Ŧ	20	ΞŪ	141070	.005	.2105
	. missing words			noi	rma l	l variate	$(\mathbf{z})$ $\mathbf{n}$	value (n	)
ομερμε: Νο	. MISSING WOLUS	(IIIW), (	equiv	1101	llia	L Vallate	mw	z z	
0090 for	block1.rng	using	hita	28	± 0	30	142029		р .6575
	block1.rng	using					141617		
	block1.rng	using					142139		
	block1.rng	using					142139		
	block1.rng	using							.2432
	-	using					142564		.9868
	block1.rng						142138		
	block1.rng	using					142211		.8468
	block1.rng	using					141872		.4497
	block1.rng	using					142263		.8847
	block1.rng	using						-1.388	.0826
	block1.rng	using					141806		.3631
	block1.rng	using						-1.154	.1243
	block1.rng	using						-1.049	.1472
	block1.rng	using					141782		.3330
	block1.rng	using					142098		.7388
	block1.rng	using					141757		.3028
	block1.rng	using					142065		.7011
	block1.rng	using					141791	401	.3442
OQSO for	block1.rng	using		10	to	14	142234	1.101	.8645
OQSO for	block1.rng	using	bits	9	to	13	141474	-1.476	.0700
OQSO for	block1.rng	using		8	to	12	142218	1.046	.8523
OQSO for	block1.rng	using	bits	7	to	11	141723	632	.2638
OQSO for	block1.rng	using	bits	6	to	10	141865	150	.4403
OQSO for	block1.rng	using	bits	5	to	9	141451	-1.554	.0601
OQSO for	block1.rng	using	bits	4	to	8	141919	.033	.5131
OQSO for	block1.rng	using			to	7	142509	2.033	.9790
	block1.rng	using			to	б	141629		.1710
	block1.rng	using			to	5	141846		.4150
	or generator bloc					-			
	. missing words		-	noi	rma]	l variate	- (z), p-	value (p	)
odopuo no		(, /	04411				mw	Z	, p
DNA for	block1.rng	using	bits	31	to	32	141889	060	.4761
	block1.rng	using					141862		.4445
	block1.rng	using					141929	.058	.5231
	block1.rng	using						-1.308	.0955
	block1.rng	using					141964		.5641
	block1.rng	using					142123		.7358
DIVES LOI	~I	ability	~CD	20	20			.050	• • • • • • •

DNA f	Eor	block1.rng	using	bits	25	to	26	141814	281	.3893
		block1.rng	using						-1.222	.1108
		block1.rng	using						.707	.7602
		block1.rng	using					141730		.2984
		block1.rng	using					142132		.7444
		block1.rng	using					142298		.8742
		block1.rng	using					142103		.7161
		block1.rng	using					142308	1.176	.8802
		block1.rng	using					141821	261	.3972
		block1.rng	using					142027	.347	.6357
		block1.rng	using					141820	264	.3961
		block1.rng	using					141978	.203	.5803
		block1.rng	using					142212	.893	.8140
		block1.rng	using					141838		.4167
		block1.rng	using					142587		.9772
		block1.rng	using					141981		.5837
		block1.rng	using					141678	682	.2475
		block1.rng	using			to	9		-1.833	.0334
		block1.rng	using			to	8	141794	340	.3669
		block1.rng	using			to	7	141596	924	.1777
		block1.rng	using			to	6	141716	570	.2842
		block1.rng	using			to	5	142093	.542	.7060
		block1.rng	using			to	4	142198	.852	.8028
		block1.rng	using		-	to	3	141994	.250	.5986
		block1.rng	using			to	2		-1.405	.0800
DNAL	LOT	DIOCKI,IIIG	using	DICS	Ŧ	ιO	2	T4T422	-1.405	.0300

This is the COUNT-THE-1's TEST on a stream of bytes. :: :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5^5 :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: ..... Test results for block1.rng Chi-square with 5^5-5^4=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block1.rng 2564.22 .908 .818123 2455.82 byte stream for block1.rng -.625 .266035 :: This is the COUNT-THE-1's TEST for specific bytes. ::

:: Consider the file under test as a stream of 32-bit integers. :: :: :: From each integer, a specific byte is chosen , say the left-:: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2406.97 -1.316 .094158 bits 2 to 9 2554.76 .774 .780651 bits 3 to 10 2421.51 -1.110.133495 bits 4 to 11 1.953 2638.13 .974619 bits 5 to 12 2575.88 1.073 .858396 bits 6 to 13 2464.64 -.500 .308537 bits 7 to 14 2351.08 -2.106 .017601 bits 8 to 15 2460.40 -.560 .287714 .050 bits 9 to 16 2503.57 .520109 bits 10 to 17 2500.53 .007 .502989 bits 11 to 18 .732 2551.73 .767771 bits 12 to 19 2413.97 -1.217.111859 bits 13 to 20 2539.52 .559 .711874 -.767 bits 14 to 21 2445.73 .221401 bits 15 to 22 2664.64 2.328 .990054 bits 16 to 23 .330 2523.30 .629128 bits 17 to 24 2459.51 -.573 .283460 bits 18 to 25 2410.58 -1.265 .103009 bits 19 to 26 2498.92 -.015 .493910 bits 20 to 27 2494.79 -.074.470634 bits 21 to 28 2504.90 .069 .527640 2337.90 bits 22 to 29 -2.292 .010940 bits 23 to 30 2486.08 -.197 .421963 bits 24 to 31 2445.35 -.773 .219815 bits 25 to 32 2524.27 .343 .634262 :: THIS IS A PARKING LOT TEST :: ::

:: In a square of side 100, randomly "park" a car---a circle of :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: time parking "by ear". That is, if an attempt to park a car :: causes a crash with one already parked, try again at a new :: random location. (To avoid path problems, consider parking :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: :: curve that should be similar to those provided by a perfect  $\colon\colon$  random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st- :: :: andard normal variable, which, converted to a uniform varia- :: :: ble, provides input to a KSTEST based on a sample of 10. :: 

CDPARK: result of ten tests on file block1.rng Of 12,000 tries, the average no. of successes

should be 3523 with sigma=21.9

Successes:	3529	z-score:	.274	p-value:	.607947
Successes:	3535	z-score:	.548	p-value:	.708135
Successes:	3527	z-score:	.183	p-value:	.572463
Successes:	3524	z-score:	.046	p-value:	.518210
Successes:	3524	z-score:	.046	p-value:	.518210
Successes:	3517	z-score:	274	p-value:	.392053
Successes:	3536	z-score:	.594	p-value:	.723613
Successes:	3505	z-score:	822	p-value:	.205562
Successes:	3519	z-score:	183	p-value:	.427537
Successes:	3516	z-score:	320	p-value:	.374623

square sizeavg. no.parkedsample sigma100.3523.2008.908KSTEST for the above 10:p=.744364

:: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the  $(n^2-n)/2$  pairs of points. If the points are truly inde- :: :: pendent uniform, then  $d^2$ , the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995 . Thus 1-exp(-d^2/.995) should be uniform on [0,1) and :: :: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 :: :: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. :: 

### This is the MINIMUM DISTANCE test

for random integers in the file block1.rng

Sample no.	d^2	avg	equiv uni
5	1.0065	1.5492	.636364
10	.2319	1.1350	.207893
15	.0331	.9154	.032708
20	1.6184	1.0570	.803390
25	4.1402	1.2587	.984408
30	1.6313	1.2451	.805924
35	1.2023	1.1596	.701304

40	3.4760	1.1402	.969604
45	.6647	1.0950	.487296
50	.3223	1.1371	.276722
55	.9412	1.0731	.611666
60	1.4237	1.0313	.760894
65	1.5687	1.0156	.793326
70	.6165	1.0044	.461818
75	2.1718	.9816	.887268
80	.0906	.9964	.086994
85	1.9667	1.0413	.861458
90	1.4027	1.0183	.755804
95	.5892	.9915	.446853
100	.7267	.9896	.518271

MINIMUM DISTANCE TEST for block1.rng Result of KS test on 20 transformed mindist^2's: p-value= .236388

THE 3DSPHERES TEST :: :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block1.rng sample no: 1 r^3= 14.966 p-value= .39278 r^3= 43.029 sample no: 2 p-value= .76172 sample no: 3 r^3= 8.329 p-value= .24242 sample no: 4 r^3= 45.484 p-value= .78044 sample no: 5 r^3= 23.237 p-value= .53910 sample no: 6 r^3= 1.065 p-value= .03486 sample no: 7
sample no: 8 r^3= 5.846 p-value= .17706 r<sup>3</sup>= 67.995 r<sup>3</sup>= 51.001 r<sup>3</sup>= 55.741 p-value= .89632 sample no: 9 p-value= .81732 sample no: 10 p-value= .84402 sample no: 11 r^3= 4.885 p-value= .15028 sample no: 12 r^3= 25.221 p-value= .56859 sample no: 13 r^3= 54.727 p-value= .83866 r^3= 3.484 sample no: 14 p-value= .10965 sample no: 15 r^3= 25.865 p-value= .57776 sample no: 16 r^3= 1.602 p-value= .05200 sample no: 17 r^3= 32.831 p-value= .66525 sample no: 18 r^3= 2.263 p-value= .07265 r^3= 40.078 p-value= .73709 p-value= .76924 sample no: 19 r^3= 43.991 sample no: 20 A KS test is applied to those 20 p-values. \_\_\_\_\_

:: This is the SQEEZE test :: Random integers are floated to get uniforms on [0,1). Start- :: :: ing with  $k=2^{31}=2147483647$ , the test finds j, the number of :: :: iterations necessary to reduce k to 1, using the reduction :: :: :: k=ceiling(k\*U), with U provided by floating integers from :: the file being tested. Such j's are found 100,000 times, :: :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$ :: :: are used to provide a chi-square test for cell frequencies. :: RESULTS OF SQUEEZE TEST FOR block1.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: -.1 .5 1.1 -.3 -.1 .4 .7 2.1 .3 -.4 .6 .1 .7 1.9 .7 -.5 -2.1 -.6 .3 -.2 -.2 1.0 -1.7 -.3 .1 -.8 -2.1 .9 -.6 .1 -.5 .6 2.1 1.2 -1.7 .0 .9 -.7 .5 -1.2 1.0 -1.0 .8 Chi-square with 42 degrees of freedom: 42.412

z-score= .045 p-value= .546769

## 

:: The OVERLAPPING SUMS test :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: :: KSTESTs are given still another KSTEST. Test no. 1 p-value .185986 p-value .704747 Test no. 2 Test no. 3 p-value .338745 Test no. 4 p-value .919336 5 p-value .820863 Test no. Test no. 6 p-value .600849 Test no. 7 p-value .973676 Test no. 8 p-value .549063 Test no. 9 p-value .335171 Test no. 10 p-value .705374 Results of the OSUM test for block1.rng KSTEST on the above 10 p-values: .602188 

:: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: :: weak inverses of the covariance matrices. Runs are counted :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file block1.rng Up and down runs in a sample of 10000

:

:

Run test for block1.rng runs up; ks test for 10 p's: .006170 runs down; ks test for 10 p's: .286105 Run test for block1.rng runs up; ks test for 10 p's: .118531 runs down; ks test for 10 p's: .287039

No. of wins:

16

334

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with :: :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: 

> Results of craps test for block1.rng Observed Expected

#### 98704 98585.86

98704= No. of wins, z-score= .528 pvalue= .70139 Analysis of Throws-per-Game: Chisq= 24.22 for 20 degrees of freedom, p= .76713 Throws Observed Expected Chisq Sum 1 66272 66666.7 2.336 2.336 2 37680 37654.3 .018 2.354 3 27082 26954.7 .601 2.955 4 19479 19313.5 1.419 4.374 5 13915 13851.4 .292 4.666 6 10046 9943.5 1.056 5.721 7 7228 7145.0 6.685 .964 5064 5139.1 8 1.097 7.782 7.789 9 3705 3699.9 .007 10 2713 2666.3 .818 8.607 11 1901 1923.3 .259 8.866 1388.7 12 1428 1.110 9.976 13 921 1003.7 6.816 16.792 14 724 726.1 16.799 .006 15 523 525.8 .015 16.814

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5.833

22.647

381.2

17	262	276.5	.764	23.411
18	202	200.8	.007	23.418
19	142	146.0	.109	23.527
20	106	106.2	.000	23.527
21	273	287.1	.694	24.221
SUMMARY F	'OR block1.	.rng		
p-valu	le for no.	of wins:	.701387	
p-valu	le for thro	ows/game:	.767132	

Results of DIEHARD battery of tests sent to file report1.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p > .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean m<sup>3</sup>/(4n). Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results ::  $\colon\colon$  to the Poisson distribution with that mean. This test uses :: :: n=2^24 and m=2^9, so that the underlying distribution for j :: :: is taken to be Poisson with lambda=2^27/(2^26)=2. A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. : : BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block2.rng For a sample of size 500: mean block2.rng using bits 1 to 24 1.962 duplicate number number spacings observed expected 0 83. 67.668 1 119. 135.335

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135.335

2

122.

3 110. 90.224 4 45.112 45. 18.045 5 18. 6 to INF 3. 8.282 .975134 Chisquare with 6 d.o.f. =14.46 p-value= For a sample of size 500: mean using bits 2 to 25 2.002 block2.rng duplicate number number spacings observed expected 62. 67.668 0 1 137. 135.335 2 140. 135.335 3 90. 90.224 4 49. 45.112 5 15. 18.045 6 to INF 8.282 7. 1.70 p-value= Chisquare with 6 d.o.f. = .055171 For a sample of size 500: mean block2.rng using bits 3 to 26 1.848 duplicate number number spacings observed expected 0 66. 67.668 1 172. 135.335 2 135.335 121. 3 90. 90.224 4 26. 45.112 5 17. 18.045 6 to INF 8. 8.282 Chisquare with 6 d.o.f. = 19.66 p-value= .996817 For a sample of size 500: mean block2.rng using bits 4 to 27 1.980 duplicate number number spacings observed expected 0 64. 67.668 1 140. 135.335 2 135.335 152. 3 69. 90.224 4 44. 45.112 5 24. 18.045 7. 6 to INF 8.282 Chisquare with 6 d.o.f. =9.60 p-value= .857237 For a sample of size 500: mean block2.rng using bits 5 to 28 1.930 duplicate number number spacings observed expected 79. 67.668 0 1 139. 135.335 2 125. 135.335 3 83. 90.224 4 50. 45.112 5 18. 18.045 6 to INF 8.282 б. Chisquare with 6 d.o.f. = 4.52 p-value= .393750

# ......

	For a sample of size 500:	mean
1	plock2.rng using bits 6 to 29	
duplicate	number number	1.900
	observed expected	
0	73. 67.668	
1	73. 67.668 131. 135.335	
2	136. 135.335	
3	90. 90.224	
4		
	40. 45.112	
5	23. 18.045	
6 to INF	7. 8.282	1 - 4 - 0 - 6
	with 6 d.o.f. = 2.70 p-value=	.154706
::::::::		
_	For a sample of size 500:	
	olock2.rng using bits 7 to 30	1.994
duplicate		
spacings	observed expected	
0	69. 67.668	
1	140. 135.335	
2	123. 135.335	
3	97. 90.224	
4	45. 45.112	
5	16. 18.045	
6 to INF	10. 8.282	
Chisquare v	with 6 d.o.f. = 2.41 p-value=	.121461
	For a sample of size 500:	mean
1	olock2.rng using bits 8 to 31	
duplicate		1.751
spacings	number number observed expected	
0	68. 67.668	
1	148. 135.335	
2	131. 135.335	
3	85. 90.224	
4	48. 45.112	
5	14. 18.045	
6 to INF		00001
chisquare \	with 6 d.o.f. = 3.35 p-value=	.2359/1
,	For a sample of size 500:	mean
	olock2.rng using bits 9 to 32	2.070
duplicate	number number	
spacings	observed expected	
0	54. 67.668	
1	126. 135.335	
2	146. 135.335	
3	109. 90.224	
4	43. 45.112	
5	16. 18.045	
6 to INF	6. 8.282	
	with 6 d.o.f. = 9.11 p-value=	.832611
	values were	
	5134 .055171 .996817 .857237	.393750
	4706 .121461 .235971 .832611	
A KSTEST :	for the 9 p-values yields .782829	

```
THE OVERLAPPING 5-PERMUTATION TEST
    ::
    :: This is the OPERM5 test. It looks at a sequence of one mill- ::
                                                           ::
    :: ion 32-bit random integers. Each set of five consecutive
    :: integers can be in one of 120 states, for the 5! possible or- ::
    :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers
                                                           ::
    :: each provide a state. As many thousands of state transitions
                                                           ::
    :: are observed, cumulative counts are made of the number of
                                                           ::
    :: occurences of each state. Then the quadratic form in the
                                                           ::
    :: weak inverse of the 120x120 covariance matrix yields a test
                                                           ::
    :: equivalent to the likelihood ratio test that the 120 cell
                                                           ::
    :: counts came from the specified (asymptotically) normal dis-
                                                           ::
    :: tribution with the specified 120x120 covariance matrix (with ::
    :: rank 99). This version uses 1,000,000 integers, twice.
                                                           ::
    OPERM5 test for file block2.rng
    For a sample of 1,000,000 consecutive 5-tuples,
chisquare for 99 degrees of freedom=107.290; p-value= .732627
         OPERM5 test for file block2.rng
    For a sample of 1,000,000 consecutive 5-tuples,
chisquare for 99 degrees of freedom= 98.253; p-value= .497661
    :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost ::
    :: 31 bits of 31 random integers from the test sequence are used ::
    :: to form a 31x31 binary matrix over the field \{0,1\}. The rank ::
    :: is determined. That rank can be from 0 to 31, but ranks< 28
                                                           ::
    :: are rare, and their counts are pooled with those for rank 28. ::
    :: Ranks are found for 40,000 such random matrices and a chisqua-::
    :: re test is performed on counts for ranks 31,30,29 and <=28.
                                                          ::
    Binary rank test for block2.rng
       Rank test for 31x31 binary matrices:
      rows from leftmost 31 bits of each 32-bit integer
          observed expected (o-e)^2/e sum
    rank
      28
              209
                     211.4
                           .027655
                                      .028
                    5134.0
      29
             5139
                           .004850
                                      .033
                                      .730
      30
            23230
                   23103.0
                           .697618
                   11551.5 1.452326
      31
            11422
                                     2.182
 chisquare= 2.182 for 3 d. of f.; p-value= .534383
_____
    :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x ::
    :: 32 binary matrix is formed, each row a 32-bit random integer. ::
    :: The rank is determined. That rank can be from 0 to 32, ranks ::
    :: less than 29 are rare, and their counts are pooled with those ::
    :: for rank 29. Ranks are found for 40,000 such random matrices ::
    :: and a chisquare test is performed on counts for ranks 32,31, ::
    :: 30 and <=29.
                                                           ::
    Binary rank test for block2.rng
       Rank test for 32x32 binary matrices:
      rows from leftmost 32 bits of each 32-bit integer
    rank
          observed expected (o-e)^2/e sum
      29
              206
                     211.4
                          .138848
                                      .139
```

30 5134.0 4.383138 4984 4.522 23103.0 31 23211 .504430 5.026 32 11599 11551.5 .195120 5.222 chisquare= 5.222 for 3 d. of f.; p-value= .854879 :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: random matrices, and a chi-square test is performed on :: counts for ranks 6,5 and <=4. Binary Rank Test for block2.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .546 r<=4 967 944.3 .546 r =5 21627 21743.9 1.174 .628 77406 77311.8 1.289 r =6 .115 p=1-exp(-SUM/2)=.47504Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .812 r<=4 972 .812 21723 21743.9 .020 .833 r =5 .001 r =6 77305 77311.8 .833 p=1-exp(-SUM/2)=.34070Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 949 944.3 .023 .023 21883 21743.9 .890 r =5 .913 77168 77311.8 .267 1.181 r = 6p=1-exp(-SUM/2)=.44587Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .332 r<=4 962 944.3 .332 r =5 21636 21743.9 .535 .867 77402 77311.8 .972 r =б .105 p=1-exp(-SUM/2)=.38504Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .454 965 944.3 .454 r<=4 21704 r =5 21743.9 .073 .527

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:: ::

::

77331 77311.8 r =6 .005 .532 p=1-exp(-SUM/2)=.23344Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .575 .575 r<=4 944.3 921 22059 21743.9 r =5 4.566 5.141 77020 77311.8 1.101 r =6 6.243 p=1-exp(-SUM/2)=.95590Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 995 944.3 2.722 2.722 r =5 21779 21743.9 .057 2.779 r =б 77226 77311.8 .095 2.874 p=1-exp(-SUM/2)=.76234Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 949 r<=4 944.3 .023 .023 .674 r =5 21865 21743.9 .698 77311.8 .205 77186 .903 r =6 p=1-exp(-SUM/2)=.36318Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .002 943 944.3 .002 r<=4 21766 .022 .024 r =5 21743.9 r =6 77291 77311.8 .006 .030 p=1-exp(-SUM/2)=.01482Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .006 r<=4 942 944.3 .006 r =5 21734 21743.9 .005 .010 77324 r =б 77311.8 .002 .012 p=1-exp(-SUM/2)=.00600Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 966 944.3 .499 .499 .526 r =5 21743.9 1.024 21637 r =6 77397 77311.8 .094 1.118 p=1-exp(-SUM/2)=.42823Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .678 .678 r<=4 919 .903 r =5 21884 21743.9 1.581 77197 77311.8 r =6 .170 1.751

p=1-exp(-SUM/2)=.58336Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .187 931 944.3 .187 r<=4 .188 .375 21680 21743.9 r =5 77389 77311.8 .077 .452 r =6 p=1-exp(-SUM/2)=.20237Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 14 to 21 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM 944.3 3.168 3.168 r<=4 999 r =5 21913 21743.9 1.315 4.483 r =б 77088 77311.8 .648 5.131 p=1-exp(-SUM/2)=.92313Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 974 944.3 .934 .934 r =5 21909 21743.9 1.254 2.188 r =6 77117 77311.8 2.678 .491 p=1-exp(-SUM/2)=.73796Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 16 to 23 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 979 944.3 1.275 1.275 21730 21743.9 .009 r =5 1.284 r =б 77291 77311.8 .006 1.290 p=1-exp(-SUM/2)=.47521Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 941 944.3 .012 r<=4 .012 21869 21743.9 r =5 .720 .731 .192 r =б 77190 77311.8 .923 p=1-exp(-SUM/2)=.36972Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 18 to 25 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 934 944.3 .112 .112 21797 .130 r =5 21743.9 .242 77269 77311.8 .024 .266 r =6 p=1-exp(-SUM/2)=.12442Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 19 to 26 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 1004 944.3 3.774 3.774 r<=4 21743.9 .537 r =5 21852 4.312 77144 77311.8 .364 4.676 r =б p=1-exp(-SUM/2)=.90347

Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 20 to 27 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 904 944.3 1.720 1.720 r =5 21822 21743.9 .281 2.001 77274 77311.8 .018 2.019 r =6 p=1-exp(-SUM/2)=.63560Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .678 919 944.3 .678 r<=4 r =5 21652 21743.9 .388 1.066 r =6 77429 77311.8 .178 1.244 p=1-exp(-SUM/2)=.46313Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 944 944.3 .000 .000 .316 r =5 21661 21743.9 .316 r =б 77395 77311.8 .090 .406 p=1-exp(-SUM/2)=.18359Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 983 944.3 1.586 1.586 r =5 21732 21743.9 .007 1.592 .009 77311.8 1.602 r =6 77285 p=1-exp(-SUM/2)=.55106Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .008 947 944.3 .008 r<=4 21491 21743.9 2.941 2.949 r =5 r =6 77562 77311.8 .810 3.759 p=1-exp(-SUM/2)=.84732Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block2.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 925 944.3 .395 .395 r =5 21771 21743.9 .034 .428 r =6 77304 77311.8 .001 .429 p=1-exp(-SUM/2)=.19309TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .475042 .340703 .445870 .385036 .233443 .955900 .762344 .363182 .014816 .006002 .202373 .428233 .583363 .923134 .737955 .369720 .903469 .635598 .475207 .124424 .183594 .193086 .463126 .551060 .847322 brank test summary for block2.rng The KS test for those 25 supposed UNI's yields

# KS p-value= .308795

:: THE BITSTREAM TEST :: :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: (j-141909)/428 should be a standard normal variate (z score) :: :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. :: THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^{21}$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ tst no 1: 142089 missing words, .42 sigmas from mean, p-value= .66268 tst no 2: 141658 missing words, -.59 sigmas from mean, p-value= .27853 141207 missing words, -1.64 sigmas from mean, p-value= .05040 tst no 3: 4: 141804 missing words, -.25 sigmas from mean, p-value= .40280 tst no tst no 5: 141515 missing words, -.92 sigmas from mean, p-value= .17844 tst no 6: 142354 missing words, 1.04 sigmas from mean, p-value= .85059 tst no 7: 141985 missing words, .18 sigmas from mean, p-value= .57017 tst no 8: 142084 missing words, .41 sigmas from mean, p-value= .65840 tst no 9: 142174 missing words, .62 sigmas from mean, p-value= .73184 tst no 10: 141404 missing words, -1.18 sigmas from mean, p-value= .11887 tst no 11: 142315 missing words, .95 sigmas from mean, p-value= .82839 141954 missing words, tst no 12: .10 sigmas from mean, p-value= .54156 142301 missing words, tst no 13: .92 sigmas from mean, p-value= .81994 141976 missing words, tst no 14: .16 sigmas from mean, p-value= .56189 -.57 sigmas from mean, p-value= .28563 141667 missing words, tst no 15: tst no 16: 141557 missing words, -.82 sigmas from mean, p-value= .20520 tst no 17: 141884 missing words, -.06 sigmas from mean, p-value= .47641 tst no 18: 141038 missing words, -2.04 sigmas from mean, p-value= .02088 tst no 19: 141555 missing words, -.83 sigmas from mean, p-value= .20387 tst no 20: 142210 missing words, .70 sigmas from mean, p-value= .75882

:: The tests OPSO, OQSO and DNA :: :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: :: "keystrokes") and counts the number of missing words---that :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with ::

:: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: consecutive bits. It then restarts the file for the next de-:: :: :: signated 10 bits, and so on. : : :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy The test OQSO is similar, except that it considers 4-letter :: : : :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: :: integers being tested. It considers 10-letter words, so that :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block2.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p)

mw  $\mathbf{z}$ р .057 OPSO for block2.rng using bits 23 to 32 141926 .5229 using bits 22 to 31 .1902 OPSO for block2.rng 141655 -.877 OPSO for block2.rng using bits 21 to 30 141540 -1.274 .1014 OPSO for block2.rng using bits 20 to 29 141766 -.494 .3106 OPSO for block2.rng using bits 19 to 28 141791 -.408 .3416 OPSO for block2.rng using bits 18 to 27 141599 -1.070 .1423 OPSO for block2.rng using bits 17 to 26 141693 -.746 .2278 OPSO for block2.rng using bits 16 to 25 142020 .6486 .382 OPSO for block2.rng using bits 15 to 24 141910 .002 .5009 OPSO for block2.rng using bits 14 to 23 142107 .682 .7523 OPSO for block2.rng using bits 13 to 22 141345 -1.946 .0258 .251 OPSO for block2.rng using bits 12 to 21 141982 .5989 OPSO for block2.rng using bits 11 to 20 142194 .982 .8369 OPSO for block2.rng using bits 10 to 19 141871 -.132 .4474 OPSO for block2.rng using bits 9 to 18 142285 1.295 .9024 OPSO for block2.rng using bits 8 to 17 142064 .533 .7031 OPSO for block2.rng using bits 7 to 16 142174 .913 .8193 OPSO for block2.rng using bits 6 to 15 141620 -.998 .1592 OPSO for block2.rng using bits .451 5 to 14 142040 .6739 OPSO for block2.rng using bits 4 to 13 -.918 141643 .1792 OPSO for block2.rng using bits 3 to 12 141938 .099 .5394 OPSO for block2.rng using bits 2 to 11 142019 .378 .6474 OPSO for block2.rng using bits 1 to 10 142118 .720 .7641 OQSO test for generator block2.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р OQSO for block2.rng using bits 28 to 32 141417 -1.669 .0476 OQSO for block2.rng using bits 27 to 31 142525 2.087 .9816

OQSO for block2.rng OQSO for block2.rng						
00S0 for block2.rng	using bits			141540 -1		.1053
- <u>-</u>	using bits	25 ta	o 29	142191	.955	.8302
OQSO for block2.rng	using bits	24 to	28	141787 -	415	.3392
OQSO for block2.rng	using bits	23 ta	o 27	142081	.582	.7197
OQSO for block2.rng	using bits	22 to	26	141832 -	262	.3966
OQSO for block2.rng	using bits	21 to	25	141808 -	343	.3656
OQSO for block2.rng	using bits			142083	.589	.7220
OQSO for block2.rng	using bits				262	.3966
OQSO for block2.rng	using bits			142059	.507	.6940
OQSO for block2.rng	using bits				479	.3159
OQSO for block2.rng	using bits				615	.2694
OQSO for block2.rng	using bits				225	.4111
OQSO for block2.rng	using bits				150	.4403
OQSO for block2.rng	using bits			142128	.741	.7707
OQSO for block2.rng	using bits				1.955	.9747
OQSO for block2.rng	using bits				1.155	.8759
OQSO for block2.rng	using bits	10 to	o 14	141631 -	943	.1727
OQSO for block2.rng	using bits	9 to	o 13	142457 1	1.857	.9683
OQSO for block2.rng	using bits	8 to	b 12	142031	.412	.6600
OQSO for block2.rng	using bits	7 to	o 11	141823 -	293	.3849
OQSO for block2.rng	using bits				L.860	.9686
OQSO for block2.rng	using bits			141521 -1		.0940
OQSO for block2.rng	using bits			141551 -1		.1122
OQSO for block2.rng	using bits			141940	.104	.5414
OQSO for block2.rng	using bits			141965	.189	.5748
	5					
OQSO for block2.rng	using bits	1 to	5 5	141535 -1	1.269	.1022
DNA test for generator			-	· · · · · · · · · · · · · · · · · · ·	· · ·	
Output: No. missing wo	rds (mw), equiv	norma	al va			)
				mw	Z	р
DNA for block2.rng	using bits				128	.4491
DNA for block2.rng	using bits			142219	.913	.8195
						.01/5
DNA for block2.rng	using bits	29 ta	o 30	141918	.026	.5102
DNA for block2.rng DNA for block2.rng	using bits using bits				.026 L.326	
		28 to	29	142359 1		.5102
DNA for block2.rng	using bits	28 ta 27 ta	29 28	142359 1 141892 -	1.326	.5102 .9077
DNA for block2.rng DNA for block2.rng	using bits using bits using bits	28 to 27 to 26 to	29 28 27	142359 1 141892 - 142418 1	1.326 051 1.501	.5102 .9077 .4796 .9333
DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits	28 to 27 to 26 to 25 to	29 28 27 26	142359 1 141892 - 142418 1 142116	1.326 051 1.501 .610	.5102 .9077 .4796 .9333 .7290
DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to	29 28 27 27 26 26 25	142359 1 141892 - 142418 1 142116 141643 -	L.326 051 L.501 .610 786	.5102 .9077 .4796 .9333 .7290 .2160
DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to	29 28 27 27 26 26 25 25 24	142359 1 141892 - 142418 1 142116 141643 - 141628 -	L.326 051 L.501 .610 786 830	.5102 .9077 .4796 .9333 .7290 .2160 .2033
DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 22 to	29 28 27 26 26 25 25 24 23	142359 1 141892 - 142418 1 142116 141643 - 141628 - 141877 -	L.326 051 L.501 .610 786 830 095	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620
DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 22 to 21 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> </ul>	142359 1 141892 - 142418 1 142116 141643 - 141628 - 141877 - 141905 -	1.326 051 1.501 .610 786 830 095 013	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949
DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to 22 to 21 to 20 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> <li>21</li> </ul>	142359 1 141892 - 142418 1 142116 141643 - 141628 - 141877 - 141905 - 142216	1.326 051 1.501 .610 786 830 095 013 .905	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172
DNA for block2.rng DNA for block2.rng	using bits using bits using bits using bits using bits using bits using bits using bits using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> <li>21</li> <li>20</li> </ul>	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 -	1.326 051 1.501 .610 786 830 095 013 .905 314	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> <li>21</li> <li>20</li> <li>21</li> <li>20</li> <li>19</li> </ul>	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 - 141599 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 17 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> <li>21</li> <li>20</li> <li>21</li> <li>20</li> <li>19</li> <li>18</li> </ul>	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 - 141599 - 141462 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915 L.320	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to	<ul> <li>29</li> <li>28</li> <li>27</li> <li>26</li> <li>25</li> <li>24</li> <li>23</li> <li>22</li> <li>21</li> <li>20</li> <li>19</li> <li>18</li> <li>17</li> </ul>	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 - 141599 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 25 to 24 to 23 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to 15 to	29         28         27         26         27         26         27         26         27         28         27         26         27         28         29         21         22         21         20         19         18         17         16	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 - 141599 - 141462 -1 142067 1	1.326 051 1.501 .610 786 830 095 013 .905 314 915 L.320	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 25 to 24 to 23 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to 15 to	29         28         27         26         27         26         27         26         27         28         27         26         27         28         29         21         22         21         20         19         18         17         16	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 141803 - 141599 - 141462 - 142067 1 142715 2	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 25 to 24 to 23 to 23 to 24 to 23 to 20 to 19 to 18 to 17 to 16 to 14 to	29         28         27         26         27         26         27         26         27         28         27         26         27         28         29         21         22         21         22         21         20         21         22         21         20         21         22         21         22         21         22         23         24         25         21         22         23         24         25         21         22         23         24         25         26         27         28         29         29         216         217         216         216         217         216	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 141803 - 141599 - 141462 - 142067 1 142715 2	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 25 to 24 to 23 to 24 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to 14 to 13 to	29         28         27         26         27         26         27         26         27         26         27         26         27         28         29         21         22         21         22         21         20         21         22         21         22         21         22         21         21         22         21         22         23         24         25         21         22         23         24         25         26         27         28         29         29         216         217         216         217         216         217         216         217         218         217	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 141803 - 141599 - 141462 -1 142067 1 142715 2 142352 1	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 23 to 24 to 23 to 20 to 19 to 19 to 18 to 16 to 14 to 13 to 21 to 20 to 21 to 20 to 21 to 22 to 23 to 24 to 23 to 24 to 25 to 24 to 25 to 24 to 25 to 24 to 25 to 24 to 25 to 24 to 25 to 24 to 20 to 21 to 20 to	29         28         27         26         27         26         27         26         27         26         27         26         27         28         27         28         27         28         27         28         29         21         22         21         22         21         22         21         22         21         22         21         22         23         24         25         21         22         23         24         25         21         22         23         24         25         26         27         28         29         29         213          29         213          29<	142359 1 141892 - 142418 1 142116 - 141643 - 141628 - 141803 - 141905 - 142216 - 142216 - 141803 - 141599 - 141462 -1 142067 - 142715 2 142352 1 142220 - 141977 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 23 to 24 to 21 to 20 to 19 to 19 to 16 to 15 to 14 to 13 to 12 to 11 to	29         28         27         26         27         26         27         26         27         26         27         28         27         26         27         28         27         28         27         28         29         21         22         21         22         21         22         21         21         22         21         22         23         24         25         21         22         23         24         25         21         22         23         24         25         26         27         28         29         213         214         22         23         24         25 <td>142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141803 - 142216 1 142216 1 142216 1 14220 1 142715 2 142220 1 142220 1 142977 1 142416 1</td> <td>1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200 1.495</td> <td>.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325</td>	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141803 - 142216 1 142216 1 142216 1 14220 1 142715 2 142220 1 142220 1 142977 1 142416 1	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200 1.495	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 23 to 22 to 21 to 20 to 19 to 16 to 13 to 12 to 11 to 10 to	29         28         27         26         27         26         27         26         27         26         27         28         27         26         27         28         27         26         27         28         29         21         22         21         22         21         20         21         22         21         22         23         24         25         21         22         23         24         25         11	142359 1 141892 - 142418 1 142116 - 141643 - 141628 - 141803 - 142216 - 142216 - 142216 - 142216 - 14267 - 142715 2 142200 - 142715 2 142220 - 141977 - 142416 - 142532 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200 1.495 1.837	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 19 to 15 to 14 to 12 to 11 to 12 to 11 to 20 to 15 to 14 to 20 to 15 to 20 to	29         28         27         26         27         26         27         26         27         28         27         28         27         28         27         28         27         28         27         28         27         28         29         21         22         21         22         21         22         21         22         21         22         23         24         25         21         22         23         24         25         11         25         11          26          27          28          29          21          21          22          23         24 <t< td=""><td>142359 141892 142418 142116 141643 141628 141877 141905 142216 141803 141599 141462 142067 142715 142352 142220 141977 142416 142532 141785</td><td>1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200 1.495 1.837 367</td><td>.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569</td></t<>	142359 141892 142418 142116 141643 141628 141877 141905 142216 141803 141599 141462 142067 142715 142352 142220 141977 142416 142532 141785	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .916 .200 1.495 1.837 367	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 15 to 14 to 14 to 12 to 11 to 20 to 20 to 15 to 14 to 20 to	$\begin{array}{c} 29\\ 28\\ 27\\ 22\\ 27\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22$	142359 141892 142418 142116 141643 141628 141877 141905 142216 141803 141599 141462 142067 142715 142352 142352 142352 142220 141977 142416 142532 141785 142192	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .200 1.495 1.837 367 .834	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569 .7978
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 14 to 14 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 17 to 18 to 17 to 19 to 19 to 10 to	$\begin{array}{c} 29\\ 28\\ 27\\ 22\\ 27\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22$	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 142067 1 142715 2 142352 1 142352 1 142532 1 142532 1 142785 - 142192 1 141734 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .200 1.495 1.837 367 .834 517	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569 .7978 .3025
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 16 to 17 to 18 to 16 to 17 to 18 to 19 to 10 to	$\begin{array}{c} 29\\ 28\\ 27\\ 22\\ 27\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22$	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 141803 - 142216 1 14267 1 14267 1 142715 2 142352 1 142352 1 142532 1 142532 1 142532 1 142785 - 142192 1 141865 -	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .200 1.495 1.837 367 .834 517 131	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569 .3569 .3569 .3925 .4480
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 22 to 20 to 19 to 18 to 17 to 16 to 14 to 13 to 14 to 12 to 14 to 12 to 14 to 15 to 14 to 15 to 14 to 5 to 5 to 5 to 5 to 25 to 26 to 27 to 29 to 20 t	$\begin{array}{c} 29\\ 28\\ 27\\ 22\\ 27\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22$	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 142067 1 14267 1 142715 2 142352 1 142220 1 142220 1 142532 1 142552 1 142555 1 142555 1 142555 1 1425555 1 1425555 1 142555555555555555555555555555555555555	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .200 1.495 1.837 367 .834 517 131 1.219	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569 .3025 .4480 .1114
DNA for block2.rng DNA for block2.rng	using bits using bits	28 to 27 to 26 to 25 to 24 to 23 to 21 to 20 to 19 to 18 to 17 to 16 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 12 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 14 to 15 to 16 to 17 to 18 to 16 to 17 to 18 to 19 to 10 to	$\begin{array}{c} 29\\ 28\\ 27\\ 22\\ 27\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22\\ 22$	142359 1 141892 - 142418 1 142116 1 141643 - 141628 - 141877 - 141905 - 142216 1 142216 1 142067 1 14267 1 142715 2 142352 1 142220 1 142220 1 142532 1 142552 1 142555 1 142555 1 142555 1 1425555 1 1425555 1 142555555555555555555555555555555555555	1.326 051 1.501 .610 786 830 095 013 .905 314 915 1.320 .465 2.377 1.306 .200 1.495 1.837 367 .834 517 131	.5102 .9077 .4796 .9333 .7290 .2160 .2033 .4620 .4949 .8172 .3769 .1800 .0935 .6791 .9913 .9042 .8203 .5791 .9325 .9669 .3569 .3569 .3569 .3925 .4480

DNA for block2.rng	using bits	3 to	4	141585	957	.1694
DNA for block2.rng	using bits	2 to	3	142239	.972	.8346
DNA for block2.rng	using bits	1 to	2	142094	.545	.7070

:: This is the COUNT-THE-1's TEST on a stream of bytes. :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5<sup>5</sup> :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: :: lapping) 5-letter words, counts are made on the frequencies :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block2.rng

Chi-square with 5^5-5^4=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block2.rng 2541.92 .593 .723350 byte stream for block2.rng 2464.41 -.503 .307360

This is the COUNT-THE-1's TEST for specific bytes. :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left- :: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. : : Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2506.72 .095 .537841

bits	2	to	9	2521.51	.304	.619530
bits	3	to	10	2565.24	.923	.821889
bits	4	to	11	2513.45	.190	.575437
bits	5	to	12	2604.65	1.480	.930567
bits	б	to	13	2495.09	069	.472321
bits	7	to	14	2468.40	447	.327491
bits	8	to	15	2497.45	036	.485615
bits	9	to	16	2495.38	065	.473977
bits	10	to	17	2606.28	1.503	.933579
bits	11	to	18	2539.53	.559	.711939
bits	12	to	19	2449.31	717	.236707
bits	13	to	20	2469.84	427	.334864
bits	14	to	21	2456.07	621	.267209
bits	15	to	22	2415.45	-1.196	.115904
bits	16	to	23	2433.21	945	.172431
bits	17	to	24	2422.24	-1.100	.135726
bits	18	to	25	2506.90	.098	.538885
bits	19	to	26	2421.74	-1.107	.134193
bits	20	to	27	2514.15	.200	.579280
bits	21	to	28	2471.12	408	.341483
bits	22	to	29	2506.29	.089	.535446
bits	23	to	30	2516.45	.233	.591973
bits	24	to	31	2577.72	1.099	.864157
bits	25	to	32	2422.23	-1.100	.135690

:: THIS IS A PARKING LOT TEST : : :: In a square of side 100, randomly "park" a car---a circle of :: :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect :: :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st- :: :: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block2.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9

Successes:	3551	z-score:	1.279	p-value:	.899470
Successes:	3513	z-score:	457	p-value:	.323972
Successes:	3519	z-score:	183	p-value:	.427537
Successes:	3504	z-score:	868	p-value:	.192812
Successes:	3580	z-score:	2.603	p-value:	.995376

Successes:	3518	z-score:	228	p-value:	.409702
Successes:	3542	z-score:	.868	p-value:	.807188
Successes:	3549	z-score:	1.187	p-value:	.882429
Successes:	3568	z-score:	2.055	p-value:	.980051
Successes:	3509	z-score:	639	p-value:	.261324
square size	avg.	no. parked	d sar	nple sigma	a
100.		3535.300	25	.066	
KSTEST for	the ab	ove 10: p=	.87666	57	

:: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the (n^2-n)/2 pairs of points. If the points are truly inde- :: :: pendent uniform, then d^2, the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995 . Thus 1-exp(-d^2/.995) should be uniform on [0,1) and :::: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 :: :: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. :: This is the MINIMUM DISTANCE test for random integers in the file block2.rng Sample no. d^2 equiv uni avg .3650 .4842 5 .307079 .7874 .699504 10 1.1963 15 2.3154 1.0809 .902413 20 1.0490 1.0153 .651555 25 1.8590 1.0995 .845628 30 4.3542 1.1730 .987425 35 1.0572 1.1370 .654400 40 .4340 1.1454 .353528 45 .1654 1.1251 .153181 50 1.2228 2.1226 .881545 1.2954 55 .5501 .424682 60 .7003 1.2717 .505286 65 1.2288 .342112 .4166 1.3118 70 .8006 .552747 75 .3872 1.2541 .322394 80 .6747 1.2308 .492421 85 .5588 1.1974 .429705 90 .0874 1.1699 .084066 95 .4232 1.1198 .346417 100 2.1494 1.1011 .884697 MINIMUM DISTANCE TEST for block2.rng Result of KS test on 20 transformed mindist^2's: p-value= .516688 

		•••
::	THE 3DSPHERES TEST	::
:: Choose	4000 random points in a cube of edge 1000. At each	::

:: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block2.rng r^3= 212.550 p-value= .99916 sample no: 1 sample no: 2 r^3= 31.020 p-value= .64442 r^3= 24.176 sample no: 3 p-value= .55330 sample no: 4 r^3= 15.143 p-value= .39635 sample no: 5 r^3= .072 p-value= .00238  $r^{3} = 59.281$   $r^{3} = 20.472$   $r^{3} = 42.123$ sample no: 6 p-value= .86138 sample no: 7 p-value= .49460 sample no: 8 p-value= .75441 sample no: 9 r^3= 14.192 p-value= .37690 r^3= .534 sample no: 10 p-value= .01765 r^3= 15.351 sample no: 11 p-value= .40052 r^3= 25.875 sample no: 12 p-value= .57790 sample no: 13 r^3= 1.325 p-value= .04320 sample no: 14 r^3= 14.000 p-value= .37292 p-value= .93294 sample no: 15 r^3= 81.065 p-value= .59937 r^3= 27.442 sample no: 16 r^3= 61.888 r^3= 23.766 p-value= .87292 sample no: 17 sample no: 18 p-value= .54715 r^3= 14.383 p-value= .38087 r^3= 13.988 p-value= .37266 sample no: 19 sample no: 20 A KS test is applied to those 20 p-values. \_\_\_\_\_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ 3DSPHERES test for file block2.rng p-value= .578757 :: This is the SQEEZE test :: :: Random integers are floated to get uniforms on [0,1). Start- :: ing with  $k=2^{31}=2147483647$ , the test finds j, the number of :: :: :: iterations necessary to reduce k to 1, using the reduction :: :: k=ceiling(k\*U), with U provided by floating integers from :: :: the file being tested. Such j's are found 100,000 times, :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$  :: :: are used to provide a chi-square test for cell frequencies. :: RESULTS OF SQUEEZE TEST FOR block2.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: 1.1 .5 -.1 .3 -.3 -.8 .3 .5 .8 .0 1.2 -1.6 .2 -1.3 .3 -.7 1.0 .0 .7 -.5 .2 .3 .3 -.2 -1.5 .9 -1.31.8 -2.3 -.5-1.5 .0 -1.2 -.3 .8 .4 -1.6 3.0 -1.8 -.7 -1.0 1.6 -1.0 -.1

Chi-square with 42 degrees of freedom: 47.267 z-score= .575 p-value= .733980

:: The OVERLAPPING SUMS test :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: Test no. 1 p-value .784409 p-value .397363 Test no. 2 Test no. 3 p-value .716635 Test no. 4 p-value .911424 Test no. 5 p-value .858380 Test no. 6 p-value .840614 Test no. 7 p-value .700915 Test no. 8 p-value .261768 Test no. 9 p-value .683462 Test no. 10 p-value .108448 Results of the OSUM test for block2.rng KSTEST on the above 10 p-values: .775137 :: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file block2.rng Up and down runs in a sample of 10000

Run test for block2.rng runs up; ks test for 10 p's: .430820 runs down; ks test for 10 p's: .632125 Run test for block2.rng runs up; ks test for 10 p's: .684980 runs down; ks test for 10 p's: .443339

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:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with : : :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: :: A chi-square test is made on the no.-of-throws cell counts. :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for block2.rng No. of wins: Observed Expected 98409 98585.86 98409= No. of wins, z-score= -.791 pvalue= .21447 Analysis of Throws-per-Game: Chisq= 15.60 for 20 degrees of freedom, p= .25885 Throws Observed Expected Chisq Sum 66322 66666.7 1.782 1.782 1 2 37654.3 37778 .406 2.188 3 27082 26954.7 .601 2.789 4 19315 19313.5 .000 2.789 5 14051 13851.4 2.876 5.665 9854 9943.5 6.471 6 .806 7 7135 7145.0 6.485 .014 5139.1 7.764 8 5058 1.279 9 3663 3699.9 .367 8.132 10 2745 2666.3 2.323 10.455 .067 11 1912 1923.3 10.521 12 1429 1388.7 11.689 1.167 1003.7 13 11.728 1010 .039 14 704 726.1 .675 12.403 15 508 525.8 .605 13.008 16 399 381.2 .836 13.844 17 286 276.5 .324 14.168 18 193 .305 14.473 200.8 19 158 146.0 .989 15.462 20 15.597 110 106.2 .135 21 288 287.1 .003 15.599 SUMMARY FOR block2.rng

p-value for no. of wins: .214467 p-value for throws/game: .258854

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Results of DIEHARD battery of tests sent to file report2.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst

in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p> .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: :: n=2^24 and m=2^9, so that the underlying distribution for j :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: :: of 500 j's is taken, and a chi-square goodness of fit test :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block3.rng For a sample of size 500: mean block3.rng using bits 1 to 24 2.042 duplicate number number spacings observed expected 0 61. 67.668 1 141. 135.335 2 132. 135.335 3 96. 90.224 4 37. 45.112 5 21. 18.045 6 to INF 12. 8.282 Chisquare with 6 d.o.f. =4.96 p-value= .450798 For a sample of size 500: mean block3.rng using bits 2 to 25 2.074 duplicate number number spacings observed expected 0 69. 67.668 1 126. 135.335 2 123. 135.335 96. 3 90.224 4 60. 45.112 5 22. 18.045 6 to INF 4. 8.282 .881871 Chisquare with 6 d.o.f. = 10.16 p-value= For a sample of size 500: mean using bits 3 to 26 1.974 block3.rng duplicate number number

observed spacings expected 67.668 0 63. 135.335 1 154. 2 130. 135.335 3 90.224 81. 4 43. 45.112 5 19. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. = 4.56 p-value= .398019 For a sample of size 500: mean using bits 4 to 27 block3.rng 1.966 duplicate number number spacings observed expected 0 63. 67.668 1 150. 135.335 2 132. 135.335 3 88. 90.224 4 45. 45.112 18.045 5 10. 6 to INF 8.282 12. Chisquare with 6 d.o.f. =7.30 p-value= .706362 For a sample of size 500: mean block3.rng using bits 5 to 28 2.016 duplicate number number expected spacings observed 0 56. 67.668 1 147. 135.335 2 135. 135.335 3 97. 90.224 4 40. 45.112 5 16. 18.045 6 to INF 9. 8.282 Chisquare with 6 d.o.f. =4.40 p-value= .377313 For a sample of size 500: mean using bits 6 to 29 block3.rng 1.994 number duplicate number spacings observed expected 72. 67.668 0 1 129. 135.335 2 133. 135.335 3 91. 90.224 4 53. 45.112 5 18.045 15. 7. 8.282 6 to INF Chisquare with 6 d.o.f. = 2.71 p-value= .156018 For a sample of size 500: mean using bits 7 to 30 block3.rng 2.062 duplicate number number spacings observed expected 0 62. 67.668 1 132. 135.335 2 132. 135.335 3 100. 90.224

4 49. 45.112 5 18.045 15. 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =2.90 p-value= .179179 For a sample of size 500: mean using bits 8 to 31 1.982 block3.rng duplicate number number spacings observed expected 0 67. 67.668 1 139. 135.335 2 135. 135.335 3 90 90.224 4 41. 45.112 5 22. 18.045 6 to INF б. 8.282 Chisquare with 6 d.o.f. = 1.98 p-value= .078257 For a sample of size 500: mean 1.968 using bits 9 to 32 block3.rng duplicate number number spacings observed expected 0 66. 67.668 135.335 1 145. 2 133. 135.335 3 91. 90.224 39. 4 45.112 5 18. 18.045 6 to INF 8. 8.282 Chisquare with 6 d.o.f. = 1.62 p-value= .048579 The 9 p-values were .450798 .881871 .398019 .706362 .377313 .156018 .179179 .078257 .048579 A KSTEST for the 9 p-values yields .727333

:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill-:: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block3.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom= 96.461; p-value= .446464 OPERM5 test for file block3.rng

For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=123.505; p-value= .951760 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block3.rng Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 5.282 211.4 5.282254 28 178 7.003 5134.0 1.720696 29 5228 30 22946 23103.0 1.067553 8.071 11648 11551.5 .805741 31 8.876 chisquare= 8.876 for 3 d. of f.; p-value= .970462 \_\_\_\_\_ :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. Binary rank test for block3.rng Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 29 268 211.4 15.143090 15.143 30 5076 5134.0 .655470 15.799 .000047 23103.0 15.799 31 23102 11554 11551.5 .000531 15.799 32 chisquare=15.799 for 3 d. of f.; p-value= .998788 \_\_\_\_\_ :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: random matrices, and a chi-square test is performed on :: :: counts for ranks 6,5 and <=4. :: Binary Rank Test for block3.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 1 to 8

OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 948 944.3 .014 .014 21954 21743.9 2.030 2.045 r =5 .591 r =б 77098 77311.8 2.636 p=1-exp(-SUM/2)=.73231Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 2 to 9 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 970 944.3 .699 .699 21850 .518 r =5 21743.9 1.217 r =6 77180 77311.8 .225 1.442 p=1-exp(-SUM/2)=.51369Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .527 .527 r<=4 922 944.3 .009 .536 21758 21743.9 r =5 77320 77311.8 .001 .537 r =6 p=1-exp(-SUM/2)=.23536Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 996 944.3 2.830 2.830 21700 21743.9 r =5 .089 2.919 r =6 77304 77311.8 .001 2.920 p=1-exp(-SUM/2)=.76774Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 951 944.3 .048 .048 21744 21743.9 .000 .048 r =5 .001 r =6 77305 77311.8 .048 p=1-exp(-SUM/2)=.02377Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 943 944.3 .002 .002 .089 .090 r =5 21700 21743.9 77357 77311.8 .026 .117 r =б p=1-exp(-SUM/2)=.05675Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .199 r<=4 958 .199 21743.9 r =5 21785 .078 .276 r =б 77257 77311.8 .039 .315 p=1-exp(-SUM/2)=.14583Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM

944.3 .006 .006 r<=4 942 21770 21743.9 r =5 .031 .037 r =6 77288 77311.8 .007 .044 p=1-exp(-SUM/2)=.02189Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 921 944.3 .575 .575 r =5 21910 21743.9 1.269 1.844 77169 .264 r =6 77311.8 2.108 p=1-exp(-SUM/2)=.65138Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 961 944.3 .295 .295 .050 .346 r =5 21777 21743.9 .378 77262 77311.8 .032 r =6 p=1-exp(-SUM/2)=.17212Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .972 .972 914 r<=4 21743.9 r =5 21844 1.433 .461 r =6 77242 77311.8 .063 1.496 p=1-exp(-SUM/2)=.52673Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 938 944.3 .042 .042 r =5 21919 21743.9 1.410 1.452 77143 r =б 77311.8 .369 1.821 p=1-exp(-SUM/2)=.59761Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .121 .121 r<=4 955 944.3 r =5 22095 21743.9 5.669 5.790 1.693 r =6 76950 77311.8 7.484 p=1-exp(-SUM/2)=.97629Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 14 to 21  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM r<=4 972 944.3 .812 .812 r =5 21674 21743.9 .225 1.037 r =6 77354 77311.8 .023 1.060 p=1-exp(-SUM/2)=.41146Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 947 944.3 .008 .008

21690 21743.9 .134 r =5 .141 r =б 77363 77311.8 .034 .175 p=1-exp(-SUM/2)=.08388Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 16 to 23  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM 910 944.3 1.246 1.246 r<=4 r =5 22038 21743.9 3.978 5.224 r =б 77052 77311.8 .873 6.097 p=1-exp(-SUM/2)=.95257Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 907 944.3 1.473 1.473 21945 21743.9 1.860 r =5 3.333 .347 r =6 77148 77311.8 3.680 p=1-exp(-SUM/2)=.84121Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 18 to 25 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 947 944.3 .008 .008 r =5 21578 21743.9 1.273 1.266 77475 77311.8 1.618 r =б .344 p=1-exp(-SUM/2)=.55469Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 19 to 26 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM .998 r<=4 975 944.3 .998 .026 r =5 21720 21743.9 1.024 r =6 77305 77311.8 .001 1.025 p=1-exp(-SUM/2)=.40096Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 20 to 27 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 993 944.3 2.511 2.511 21946 21743.9 1.878 r =5 4.390 77061 77311.8 .814 5.204 r =6 p=1-exp(-SUM/2)=.92586Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .998 r<=4 975 944.3 .998 21743.9 r =5 21836 .390 1.388 77189 77311.8 r =б .195 1.583 p=1-exp(-SUM/2)=.54687Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 986 944.3 1.841 r<=4 1.841 21867 r =5 21743.9 .697 2.538

77147 77311.8 2.890 r =6 .351 p=1-exp(-SUM/2)=.76420Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 951 944.3 .048 .048 21743.9 .177 21797 .130 r =5 77311.8 r =6 77252 .046 .223 p=1-exp(-SUM/2)=.10571Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 940 944.3 .020 .020 21777 r =5 21743.9 .050 .070 r =б 77283 77311.8 .011 .081 p=1-exp(-SUM/2)=.03955Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block3.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 942 r<=4 944.3 .006 .006 .039 r =5 21773 21743.9 .045 77285 77311.8 .009 r =б .054 p=1-exp(-SUM/2)=.02656TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .732309 .513686 .235357 .767744 .023772 .145834 .056750 .021890 .651385 .172116 .597610 .976289 .411458 .526727 .083885 .554691 .952568 .841214 .400965 .925856 .039552 .546874 .764203 .105711 .026563 brank test summary for block3.rng The KS test for those 25 supposed UNI's yields KS p-value= .805665

THE BITSTREAM TEST :: :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... . Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. : : THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^{21}$  and samples the bitstream 20 times.

No. missing words should average 141909. with sigma=428.

tst	no	1:	141543	missing	words,	86	sigmas	from m	nean,	p-value=	.19602
tst	no	2:	142352	missing	words,	1.03	sigmas	from m	nean,	p-value=	.84950
tst	no	3:	141698	missing	words,	49	sigmas	from m	nean,	p-value=	.31074
tst	no	4:	141445	missing	words,	-1.08	sigmas	from m	nean,	p-value=	.13899
tst	no	5:	142411	missing	words,	1.17	sigmas	from m	nean,	p-value=	.87943
tst	no	6:	142308	missing	words,	.93	sigmas	from m	nean,	p-value=	.82420
tst	no	7:	141282	missing	words,	-1.47	sigmas	from m	nean,	p-value=	.07136
tst	no	8:	142059	missing	words,	.35	sigmas	from m	nean,	p-value=	.63672
tst	no	9:	141818	missing	words,	21	sigmas	from m	nean,	p-value=	.41551
tst	no	10:	141039	missing	words,	-2.03	sigmas	from m	nean,	p-value=	.02100
tst	no	11:	141849	missing	words,	14	sigmas	from m	nean,	p-value=	.44395
tst	no	12:	141853	missing	words,	13	sigmas	from m	nean,	p-value=	.44765
tst	no	13:	141787	missing	words,	29	sigmas	from m	nean,	p-value=	.38751
tst	no	14:	141854	missing	words,	13	sigmas	from m	nean,	p-value=	.44857
tst	no	15:	142041	missing	words,	.31	sigmas	from m	nean,	p-value=	.62082
tst	no	16:	142027	missing	words,	.27	sigmas	from m	nean,	p-value=	.60832
tst	no	17:	142106	missing	words,	.46	sigmas	from m	nean,	p-value=	.67707
tst	no	18:	141772	missing	words,	32	sigmas	from m	nean,	p-value=	.37416
tst	no	19:	142356	missing	words,	1.04	sigmas	from m	nean,	p-value=	.85167
tst	no	20:	141874	missing	words,	08	sigmas	from m	nean,	p-value=	.46711

:: The tests OPSO, OQSO and DNA :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: :: integers being tested. It considers 10-letter words, so that :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. ::

:: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block3.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw  $\mathbf{Z}$ р 142294 1.326 .9077 OPSO for block3.rng using bits 23 to 32 OPSO for block3.rng using bits 22 to 31 141830 -.274 .3922 OPSO for block3.rng using bits 21 to 30 141767 -.491 .3118 OPSO for block3.rng using bits 20 to 29 141966 .195 .5775 OPSO for block3.rng using bits 19 to 28 142436 1.816 .9653 OPSO for block3.rng using bits 18 to 27 141697 -.732 .2320 OPSO for block3.rng using bits 17 to 26 141747 -.560 .2878 OPSO for block3.rng using bits 16 to 25 141758 -.522 .3009 OPSO for block3.rng using bits 15 to 24 141935 .089 .5353 .020 OPSO for block3.rng using bits 14 to 23 141915 .5078 OPSO for block3.rng using bits 13 to 22 141910 .002 .5009 .113 OPSO for block3.rng using bits 12 to 21 141942 .5449 -.491 OPSO for block3.rng using bits 11 to 20 141767 .3118 OPSO for block3.rng using bits 10 to 19 141807 -.353 .3621 OPSO for block3.rng using bits 9 to 18 141335 -1.980 .0238 OPSO for block3.rng using bits 8 to 17 141511 -1.374 .0848 OPSO for block3.rng using bits 7 to 16 141758 -.522 .3009 -.022 OPSO for block3.rng using bits 6 to 15 141903 .4913 OPSO for block3.rng using bits 5 to 14 142145 .813 .7918 OPSO for block3.rng using bits 4 to 13 142016 .368 .6435 OPSO for block3.rng using bits 3 to 12 142096 .644 .7401 OPSO for block3.rng using bits 2 to 11 142079 .585 .7208 OPSO for block3.rng using bits 1 to 10 141938 .099 .5394 OQSO test for generator block3.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р OQSO for block3.rng using bits 28 to 32 141980 .240 .5947 .114 OQSO for block3.rng using bits 27 to 31 141943 .5454 .782 OQSO for block3.rng using bits 26 to 30 142140 .7829 OQSO for block3.rng using bits 25 to 29 .460 .6772 142045 OQSO for block3.rng using bits 24 to 28 141602 -1.042 .1488 using bits 23 to 27 OQSO for block3.rng 141965 .189 .5748 OQSO for block3.rng using bits 22 to 26 141830 -.269 .3940 OQSO for block3.rng using bits 21 to 25 141683 -.767 .2215 OQSO for block3.rng using bits 20 to 24 141580 -1.116 .1321 OQSO for block3.rng using bits 19 to 23 142123 .724 .7656 OQSO for block3.rng using bits 18 to 22 142102 .653 .7432 OQSO for block3.rng using bits 17 to 21 141844 -.221 .4124 OQSO for block3.rng using bits 16 to 20 142382 1.602 .9455 OQSO for block3.rng using bits 15 to 19 141566 -1.164 .1222 OQSO for block3.rng using bits 14 to 18 -.137 141869 .4456 using bits 13 to 17 OQSO for block3.rng .006 141911 .5023 using bits 12 to 16 OQSO for block3.rng 142176 .904 .8170 OQSO for block3.rng using bits 11 to 15 142000 .307 .6207 OQSO for block3.rng using bits 10 to 14 141814 -.323 .3733 OQSO for block3.rng using bits 9 to 13 142282 1.263 .8968 .843 OQSO for block3.rng using bits 8 to 12 142158 .8004 using bits OQSO for block3.rng 7 to 11 142002 .314 .6233 6 to 10 141375 -1.811 OQSO for block3.rng using bits .0350 OQSO for block3.rng using bits 5 to 9 141745 -.557 .2887

~		block3.rng	using			to	8	141719	645	.2594
OQSO	for	block3.rng	using	bits	3	to	7	141992		.6104
OQSO	for	block3.rng	using	bits	2	to	6	141966	.192	.5762
OQSO	for	block3.rng	using	bits	1	to	5	142118	.707	.7603
		or generator blo								
Output:	No	. missing words	(mw), e	equiv	noi	rmal	l variate			)
								mw	Z	р
		block3.rng	using					141796	334	.3691
		block3.rng	using					141743		
		block3.rng	using					141622		
		block3.rng	using					141643		
		block3.rng	using					141869		
DNA	for	block3.rng	using	bits	26	to	27	142194	.840	
DNA	for	block3.rng	using	bits	25	to	26	142230	.946	.8279
		block3.rng	using	bits	24	to	25	141925	.046	.5184
DNA	for	block3.rng	using	bits	23	to	24	141860	146	.4422
DNA	for	block3.rng	using	bits	22	to	23	141837	213	.4155
DNA	for	block3.rng	using	bits	21	to	22	141860	146	.4422
DNA	for	block3.rng	using	bits	20	to	21	141683	668	.2522
DNA	for	block3.rng	using	bits	19	to	20	142454	1.607	.9459
DNA	for	block3.rng	using	bits	18	to	19	141938	.085	.5337
DNA	for	block3.rng	using	bits	17	to	18	141661	733	.2319
DNA	for	block3.rng	using	bits	16	to	17	141546	-1.072	.1419
DNA	for	block3.rng	using	bits	15	to	16	141922	.037	.5149
DNA	for	block3.rng	using	bits	14	to	15	141922	.037	.5149
		block3.rng	using					142113	.601	.7260
		block3.rng	using					141464	-1.314	.0945
		block3.rng	using						-1.010	.1563
		block3.rng	using					141957	.141	.5559
		block3.rng	using			to		141672		.2419
		block3.rng	using			to	9	141588	948	.1716
		block3.rng	using			to	8	142070	.474	.6822
		block3.rng	using			to	7	142452	1.601	.9453
		block3.rng	using			to	6	142150	.710	.7611
		block3.rng	using			to	5	141867		.4503
	-	block3.rng	using			to	4	142282	1.099	.8642
		block3.rng	using		-	to	3	141848	181	.4282
		block3.rng	using			to	2		-2.134	.0164
DINA	TOT	2100100.1119	abilig	2100	-	20	-	± 1±±00	2.131	.0101

:: This is the COUNT-THE-1's TEST on a stream of bytes. :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5^5  $\,$ :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of ::

:: This is the COUNT-THE-1's TEST for specific bytes. :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left- :: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. : : Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2502.17 .031 .512240 bits 2 to 9 2391.41 -1.536 .062312 bits 3 to 10 -.306 .379749 2478.35 bits 4 to 11 2461.14 -.550 .291325 bits 5 to 12 2466.26 -.477 .316638 bits 6 to 13 2515.79 .223 .588350 bits 7 to 14 2574.38 1.052 .853571 bits 8 to 15 2595.48 1.350 .911533 bits 9 to 16 2413.19 -1.228 .109779 bits 10 to 17 2515.39 .218 .586149 bits 11 to 18 2474.26 -.364 .357912 .223429 bits 12 to 19 2446.21 -.761 bits 13 to 20 2516.80 .238 .593913 bits 14 to 21 2527.37 .387 .650665 bits 15 to 22 2476.58 -.331 .370267 bits 16 to 23 2406.41 -1.324 .092826 bits 17 to 24 2445.97 -.764 .222393 bits 18 to 25 2508.31 .118 .546792 .309 bits 19 to 26 2521.86 .621399 -.133 .446987 bits 20 to 27 2490.58 bits 21 to 28 2561.37 .868 .807263 bits 22 to 29 2470.35 -.419 .337491 bits 23 to 30 2596.10 1.359 .912933

bits	24	to	31	2476.07	338	.367532
bits	25	to	32	2450.33	702	.241220

:: THIS IS A PARKING LOT TEST :: :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect :: :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st-:: :: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block3.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 Successes: 3510 z-score: -.594 p-value: .276387 Successes: 3521 z-score: -.091 p-value: .463618 Successes: 3550 z-score: 1.233 p-value: .891189 Successes: 3519 z-score: -.183 p-value: .427537 -.183 p-value: .427537 Successes: 3519 z-score: Successes: 3515 -.365 p-value: .357445 z-score: Successes: 3506 -.776 p-value: .218799 z-score: Successes: 3552 z-score: 1.324 p-value: .907282 Successes: 3532 .411 p-value: .659449 z-score: Successes: 3514 z-score: -.411 p-value: .340551 square size avg. no. parked sample sigma 100. 3523.800 15.072 KSTEST for the above 10: p= .451665 :: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a ::

:: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the (n^2-n)/2 pairs of points. If the points are truly inde- :: :: pendent uniform, then d^2, the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995 . Thus 1-exp(-d^2/.995) should be uniform on [0,1) and :: :: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 ::

:: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. :: This is the MINIMUM DISTANCE test for random integers in the file block3.rng Sample no. d^2 avg equiv uni .3566 .301211 1.2487 5 10 .2498 1.2662 .222060 15 2.2682 1.1231 .897670 .203071 20 .2259 1.1401 25 .3029 1.1136 .262461 30 .7571 1.0964 .532772 1.0675 35 .8437 .571694 40 1.1572 1.1584 .687463 45 .2061 1.1185 .187072 50 1.3004 1.0809 .729338 55 .4219 1.0482 .345610 .4818 1.0364 60 .383794 1.0061 .402419 65 .5123 .9850 70 1.1150 .673903 .9704 75 .4943 .391531 80 1.8742 .9571 .847955 85 .9553 .1597 .148269 2.0265 .869536 90 .9646 95 1.3205 .9608 .734760 100 .6081 .9716 .457292 MINIMUM DISTANCE TEST for block3.rng Result of KS test on 20 transformed mindist^2's: p-value= .211625 :: THE 3DSPHERES TEST :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block3.rng sample no: 1 r^3= 42.873 p-value= .76048  $r^{3} = 25.552$ sample no: 2 p-value= .57333 r^3= 24.719 sample no: 3 p-value= .56131 sample no: 4 r^3= 5.416 p-value= .16517

10 0111 <u>1</u> = 0		_			T	
sample	no:	5	r^3=	15.757	p-value=	.40858
sample	no:	6	r^3=	43.326	p-value=	.76406
sample	no:	7	r^3=	2.055	p-value=	.06621
sample	no:	8	r^3=	37.658	p-value=	.71500
sample	no:	9	r^3=	12.371	p-value=	.33792
sample	no:	10	r^3=	30.587	p-value=	.63924
sample	no:	11	r^3=	17.624	p-value=	.44426
sample	no:	12	r^3=	21.512	p-value=	.51182

r^3= .120 p-value= .00398 r^3= 18.439 p-value= .45917 r^3= 14.804 p-value= .38949 r^3= 32.237 p-value= .65856 r^3= 86.750 p-value= .94452 r^3= 6.576 p-value= .19683 sample no: 13 sample no: 14 sample no: 15 sample no: 16 sample no: 17 sample no: 18 r<sup>3</sup>= 5.668 p-value= .17216 r<sup>3</sup>= 8.877 p-value= .25614 sample no: 19 p-value= .25614 sample no: 20 A KS test is applied to those 20 p-values. -----3DSPHERES test for file block3.rng p-value= .411318 :: This is the SQEEZE test :: :: Random integers are floated to get uniforms on [0,1). Start- :: :: ing with k=2^31=2147483647, the test finds j, the number of :: :: iterations necessary to reduce k to 1, using the reduction :: :: k=ceiling(k\*U), with U provided by floating integers from :: :: the file being tested. Such j's are found 100,000 times, :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$  :: :: are used to provide a chi-square test for cell frequencies. :: RESULTS OF SQUEEZE TEST FOR block3.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: .1 .6 -.8 -.7 .2 1.2 .9 -.6 -.8 -1.3 -.б -.1 -.6 .0 -1.0 -.2 1.5 .0 .2 .7 .0 -1.1 .3 -1.0 .6 2.0 .8 -.5 .6 1.1 2.1 .8 -.2 .1 .7 -.5 .9 -1.6 -1.3 -.7 -.1 2.0 .8 Chi-square with 42 degrees of freedom: 36.324 z-score= -.619 p-value= .282057

## 

Test no. 5

The OVERLAPPING SUMS test :: :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: Test no. 1 p-value .864680 Test no. 2 p-value .519832 Test no. 3 p-value .883525 Test no. 4 p-value .860066

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p-value .519336

Test no. 6 p-value .526971 7 Test no. p-value .842970 Test no. 8 p-value .611857 Test no. 9 p-value .540477 Test no. 10 p-value .350981 Results of the OSUM test for block3.rng KSTEST on the above 10 p-values: .913041

:: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file block3.rng

:

Up and down runs in a sample of 10000

Run test for block3.rng runs up; ks test for 10 p's: .414940 runs down; ks test for 10 p's: .276074 Run test for block3.rng runs up; ks test for 10 p's: .695511 runs down; ks test for 10 p's: .206234

## 

2

3

37705

27050

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with :: :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for block3.rng No. of wins: Observed Expected 98657 98585.86 98657= No. of wins, z-score= .318 pvalue= .62483 Analysis of Throws-per-Game: Chisq= 27.83 for 20 degrees of freedom, p= .88641 Throws Observed Expected Chisq Sum 1 66161 66666.7 3.836 3.836

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.068

.337

3.904

4.240

37654.3

26954.7

4	19665	19313.5	6.399	10.639
5	13731	13851.4	1.047	11.686
б	9942	9943.5	.000	11.686
7	7167	7145.0	.068	11.754
8	5138	5139.1	.000	11.754
9	3800	3699.9	2.710	14.464
10	2669	2666.3	.003	14.467
11	1976	1923.3	1.442	15.909
12	1391	1388.7	.004	15.913
13	971	1003.7	1.066	16.979
14	666	726.1	4.981	21.960
15	532	525.8	.072	22.033
16	411	381.2	2.338	24.370
17	268	276.5	.264	24.634
18	203	200.8	.023	24.657
19	166	146.0	2.744	27.402
20	110	106.2	.135	27.536
21	278	287.1	.289	27.826
SUMMARY	FOR bloc	k3.rng		

p-value for no. of wins: .624827
p-value for throws/game: .886415

#### 

Results of DIEHARD battery of tests sent to file report3.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p> .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: is taken to be Poisson with lambda=2^27/(2^26)=2. A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: :: provides a p value. The first test uses bits 1-24 (counting :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are ::

:: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block4.rng For a sample of size 500: mean using bits 1 to 24 2.042 block4.rng duplicate number number spacings observed expected 69. 67.668 0 1 147. 135.335 2 104. 135.335 3 107. 90.224 4 38. 45.112 5 23. 18.045 6 to INF 12. 8.282 Chisquare with 6 d.o.f. = 15.56 p-value= .983664 For a sample of size 500: mean block4.rng using bits 2 to 25 1.932 duplicate number number spacings observed expected 0 75. 67.668 1 135.335 136. 2 134. 135.335 3 90.224 95. 4 35. 45.112 5 16. 18.045 6 to INF 9. 8.282 Chisquare with 6 d.o.f. = 3.62 p-value= .272641 For a sample of size 500: mean block4.rng using bits 3 to 26 1.920 duplicate number number spacings observed expected 67.668 0 76. 1 135.335 140. 2 135. 135.335 3 78. 90.224 4 52. 45.112 5 9. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =8.79 p-value= .813994 For a sample of size 500: mean block4.rng using bits 4 to 27 2.114 duplicate number number spacings observed expected 0 58. 67.668 1 122. 135.335 2 143. 135.335 3 102. 90.224 4 50. 45.112 5 15. 18.045 10. 6 to INF 8.282 Chisquare with 6 d.o.f. = 6.07 p-value= .584197

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1- 1	FOr a sa	ample of size 500:	illean
	LOCK4.rng	using bits 5 to 28	1.988
duplicate	number	number	
	observed		
0	58.	67.668	
1	150.	135.335	
2	136.	135.335	
3	85.	90.224	
4		45.112	
5	13.	18.045	
6 to INF	9.	8.282	
Chisquare wi	th 6 d.o.f.	= 5.08 p-value=	.466874
:::::::::::			
	For a sa	ample of size 500:	mean
bl		using bits 6 to 29	
duplicate	number		
	observed		
0	57.	67.668	
1	148.	135.335	
2	132.	135.335	
3		90.224	
4		45.112	
5		18.045	
6 to INF	б.	8.282	
		= 7.97 p-value=	.759703
:::::::::::			
		ample of size 500:	
		using bits 7 to 30	1.950
duplicate	number	number	
spacings	observed	expected	
0	72.	67.668	
1	130.	135.335	
2	140.	135.335	
3	100.		
4	35.	45.112	
5	14.	18.045	
6 to INF	9.	8.282	
Chigguare wi	th 6d of	= 4.94 p-value=	448894
			. 110091
		ample of size 500:	mean
ы	lock4.rng	using bits 8 to 31	
	number	number	2.024
duplicate			
spacings	observed	expected	
0	65.	67.668	
1	141.	135.335	
2	126.	135.335	
3	88.	90.224	
4	55.	45.112	
5	18.	18.045	
6 to INF	7.	8.282	
Chisquare wi	th 6 d.o.f.	= 3.41 p-value=	.243695
::::::::::			
	For a sa	ample of size 500:	mean
bl	.ock4.rng	using bits 9 to 32	
duplicate	number	number	
spacings	observed	expected	
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0 56. 67.668 135.335 134. 1 2 141. 135.335 3 86. 90.224 4 48. 45.112 5 24. 18.045 11. 8.282 6 to INF Chisquare with 6 d.o.f. = 5.50 p-value= .518819 The 9 p-values were .983664 .272641 .813994 .584197 .466874 .759703 .448894 .243695 .518819 A KSTEST for the 9 p-values yields .373832

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:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block4.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=106.939; p-value= .724796 OPERM5 test for file block4.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom= 98.789; p-value= .512905 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block4.rng Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer observed expected  $(o-e)^2/e$  sum rank .099 2.8 216 211.4 .099304 5134.0 1.246908 29 5054 1.346 .029366 1.376 30 23077 23103.0 31 11653 11551.5 .891423 2.267 chisquare= 2.267 for 3 d. of f.; p-value= .547206 \_\_\_\_\_ 

:: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. :: Binary rank test for block4.rng Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer observed expected (o-e)^2/e sum rank 29 220 211.4 .348364 .348 30 5067 5134.0 .874633 1.223 31 23016 23103.0 .327972 1.551 11551.5 1.832065 32 11697 3.383 chisquare= 3.383 for 3 d. of f.; p-value= .696684

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:: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: random matrices, and a chi-square test is performed on :: :: counts for ranks 6,5 and <=4. : : Binary Rank Test for block4.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 1 to 8  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM r<=4 997 944.3 2.941 2.941 21793 .111 r =5 21743.9 3.052 r =6 77210 77311.8 .134 3.186 p=1-exp(-SUM/2)=.79667Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r < = 4952 944.3 .063 .063 r =5 21798 21743.9 .135 .197 r =6 77250 77311.8 .049 .247 p=1-exp(-SUM/2)=.11608Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .248 .248 r<=4 929 944.3 r =5 22135 21743.9 7.035 7.283 77311.8 1.827 76936 9.109 r =6 p=1-exp(-SUM/2)=.98948Rank of a 6x8 binary matrix,

rows formed from eight bits of the RNG block4.rng b-rank test for bits 4 to 11 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r <= 4985 944.3 1.754 1.754 22045 21743.9 r =5 4.169 5.924 76970 77311.8 1.511 7.435 r =6 p=1-exp(-SUM/2)=.97570Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 979 944.3 1.275 1.275 .349 r =5 21831 21743.9 1.624 77190 77311.8 .192 1.816 r =б p=1-exp(-SUM/2)=.59663Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 959 944.3 .229 r<=4 .229 21977 2.728 r =5 21743.9 2.499 77064 77311.8 .794 3.522 r =6 p=1-exp(-SUM/2)=.82812Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 999 944.3 3.168 3.168 r =5 21794 21743.9 .115 3.284 .142 r =6 77207 77311.8 3.426 p=1-exp(-SUM/2)=.81967Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 902 944.3 1.895 1.895 r<=4 r =5 21537 21743.9 1.969 3.864 77561 77311.8 r =б .803 4.667 p=1-exp(-SUM/2)=.90304Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 898 944.3 2.270 2.270 r<=4 r =5 21536 21743.9 1.988 4.258 r =6 77566 77311.8 .836 5.094 p=1-exp(-SUM/2)=.92168Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r <= 4979 944.3 1.275 1.275 21540 21743.9 r =5 1.912 3.187 .370 77481 77311.8 3.557 r =6 p=1-exp(-SUM/2)=.83114Rank of a 6x8 binary matrix,

rows formed from eight bits of the RNG block4.rng

b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 981 944.3 1.426 1.426 r =5 21722 21743.9 .022 1.448 77297 77311.8 .003 r =6 1.451 p=1-exp(-SUM/2)=.51595Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 920 944.3 .625 .625 r =5 21889 21743.9 .968 1.594 r =6 77191 77311.8 .189 1.782 p=1-exp(-SUM/2)=.58984Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 3.009 3.009 891 r<=4 4.759 21939 21743.9 1.751 r =5 77170 r =6 77311.8 .260 5.019 p=1-exp(-SUM/2)=.91870Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 14 to 21 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 887 944.3 3.477 3.477 r =5 21797 21743.9 .130 3.607 r =6 77316 77311.8 .000 3.607 p=1-exp(-SUM/2)=.83528Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 986 944.3 1.841 1.841 .144 r =5 21688 21743.9 1.985 77326 77311.8 .003 1.988 r =6 p=1-exp(-SUM/2)=.62984Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 16 to 23 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 989 944.3 2.116 2.116 5.397 22011 21743.9 3.281 r =5 r =6 77000 77311.8 1.258 6.654 p=1-exp(-SUM/2)=.96411Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 995 944.3 2.722 2.722 r =5 21909 21743.9 1.254 3.976 .602 r =6 77096 77311.8 4.578 p=1-exp(-SUM/2)= .89863 Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 18 to 25

EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 947 944.3 .008 .008 21764 21743.9 r =5 .019 .026 r =б 77289 77311.8 .007 .033 p=1-exp(-SUM/2)=.01637Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 19 to 26 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 966 944.3 .499 .499 21799 r =5 21743.9 .140 .638 r =6 77235 77311.8 .076 .715 p=1-exp(-SUM/2)=.30041Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 20 to 27 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 893 944.3 2.787 2.787 21930 1.593 21743.9 4.380 r =5 77177 77311.8 .235 4.615 r =6 p=1-exp(-SUM/2)=.90048Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 924 944.3 .436 .436 21743.9 r =5 21827 .318 .754 r =6 77249 77311.8 .051 .805 p=1-exp(-SUM/2)=.33137Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 953 944.3 .080 .080 21853 21743.9 .547 .628 r =5 .180 r =6 77194 77311.8 .807 p=1-exp(-SUM/2)=.33204Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 974 944.3 .934 .934 .031 .965 r =5 21770 21743.9 77256 77311.8 .040 1.006 r =6 p=1-exp(-SUM/2)=.39518Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 924 944.3 r<=4 .436 .436 21743.9 r =5 21633 .566 1.002 r =б 77443 77311.8 .223 1.225 p=1-exp(-SUM/2)=.45793Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block4.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM

<pre>rc=4 974 944.3 .934 .934 r=5 21564 21743.9 1.488 2.422 r=6 77462 77311.8 .292 2.714 p=1-exp(-SUM/2)=.74260 TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .796674 .116079 .989482 .975702 .596633 .828123 .819670 .90303 .921677 .831138 .515949 .599840 .918701 .835279 .629844 .96616 .8996630 .016374 .300413 .900484 .331373 .332036 .395180 .457927 .742600 brank test summary for block4.rng The KS test for those 25 supposed UNI's yields KS p-value= .995650 SS\$</pre>							
<ul> <li>r = 6 77462 77311.8 .292 2.714 p=1-exp(-SUM/2)= 774260</li> <li>TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices</li> <li>These should be 25 uniform [0,1] random variables: .796674 .116079 .998482 .975702 .596633</li> <li>.828123 .819670 .903039 .921677 .831138</li> <li>.51594 .518940 .918701 .335279 .629844</li> <li>.964106 .898630 .016374 .300413 .900484</li> <li>.33137 .332026 .395180 .457927 .742600</li> <li>brank test summary for block4.rng</li> <li>The KS test for those 25 supposed UNI's yields KS p-value= .995650</li> <li>\$</li></ul>							
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<pre>:: THE BITSTREAM TEST :: :: The file under test is viewed as a stream of bits. Call them :: :: bl,b2, Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is bb2b20, the :: :: second is bb3b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: :: (j-14190)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. :: :: twenty times. :: :: twenty times. :: :: :: twenty times. :: :: tst no 1: 141285 missing words, -1.46 sigmas from mean, p-value= .07232 tst no 2: 141829 missing words, -1.00 sigmas from mean, p-value= .15960 tst no 3: 141483 missing words, -1.00 sigmas from mean, p-value= .42556 tst no 3: 141480 missing words, -0.5 sigmas from mean, p-value= .48199 tst no 6: 142712 missing words, -0.5 sigmas from mean, p-value= .48199 tst no 6: 142712 missing words, -1.21 sigmas from mean, p-value= .48199 tst no 6: 142712 missing words, -1.21 sigmas from mean, p-value= .48199 tst no 6: 142712 missing words, -1.21 sigmas from mean, p-value= .48458 tst no 1: 141266 missing words, -1.21 sigmas from mean, p-value= .48458 tst no 1: 14126 missing words, -1.11 sigmas from mean, p-value= .48458 tst no 1: 14126 missing words, -1.13 sigmas from mean, p-value= .48458 tst no 1: 14122 missing words, -1.13 sigmas from mean, p-value= .48458 tst no 11: 14216 missing words, -0.5 sigmas from mean, p-value= .48458 tst no 11: 14216 missing words, -0.15 sigmas from mean, p-value= .48458 tst no 11: 14216 missing words, -0.15 sigmas from mean, p-value= .48458 tst no 11: 14216 missing words, -1.15 sigmas from mean, p-value= .48458</pre>		KS p-val	ue= .995650				
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tst no2:141829 missing words, missing words,19 sigmas from mean, p-value=.42556tst no3:141483 missing words, missing words,100 sigmas from mean, p-value=.15960tst no4:141640 missing words, missing words,63 sigmas from mean, p-value=.26459tst no5:141890 missing words, missing words,05 sigmas from mean, p-value=.26459tst no6:142712 missing words, missing words,05 sigmas from mean, p-value=.6458tst no7:142068 missing words, missing words,111 sigmas from mean, p-value=.64458tst no9:141436 missing words, missing words,121 sigmas from mean, p-value=.11249tst no11:141522 missing words, missing words,90 sigmas from mean, p-value=.12297tst no11:142186 missing words, missing words,115 sigmas from mean, p-value=.12597tst no12:141419 missing words, missing words,14 sigmas from mean, p-value=.12890tst no14:142004 missing words, missing words,13 sigmas from mean, p-value=.12890tst no15:141825 missing words, missing words,73 sigmas from mean, p-value=.24192tst no18:141883 missing words, missing words,06 sigmas from mean, p-value=.2425773 sigmas from mean, p-value=.2425806 sigmas from mean, p-value=.2425773 sigmas from mean, p-value=.24192.2313506 sigmas from mean, p-	tat no 1.	 141285 migging	words $-1$ 46		from mean		07232
tst no3:141483 missing words, tst no-1.00 sigmas from mean, p-value=.15960tst no4:141640 missing words, tst no63 sigmas from mean, p-value=.26459tst no5:141890 missing words, tst no05 sigmas from mean, p-value=.48199tst no6:142712 missing words, tst no1.88 sigmas from mean, p-value=.96963tst no7:142068 missing words, tst no-1.21 sigmas from mean, p-value=.64458tst no9:141436 missing words, tst no-1.11 sigmas from mean, p-value=.11249tst no11:142186 missing words, tst no90 sigmas from mean, p-value=.18274tst no12:141419 missing words, tst no115 sigmas from mean, p-value=.12597tst no13:141850 missing words, tst no14 sigmas from mean, p-value=.26459tst no14:142004 missing words, tst no05 sigmas from mean, p-value=.64458tst no11:142186 missing words, tst no121 sigmas from mean, p-value=.1249tst no13:141850 missing words, tst no14 sigmas from mean, p-value=.26459tst no14:142004 missing words, tst no121 sigmas from mean, p-value=.1249tst no14:142004 missing words, tst no13 sigmas from mean, p-value=.25753tst no16:142197 missing words, tst no73 sigmas from mean, p-value=.24925.58.58sigmas from mean, p-value= </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>							
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tst no 8: 141390 missing words, tst no 9: 141436 missing words, tst no 10: 141522 missing words, tst no 11: 142186 missing words, tst no 12: 141419 missing words, tst no 13: 141850 missing words, tst no 14: 142004 missing words, tst no 15: 141425 missing words, tst no 16: 142197 missing words, tst no 18: 141833 missing words, tst no 19: 141233 missing words, tst no 19: 141233 missing words, tst no 19: 141233 missing words,							
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<ul> <li>tst no 10: 141522 missing words,</li> <li>tst no 11: 142186 missing words,</li> <li>tst no 12: 141419 missing words,</li> <li>tst no 13: 141850 missing words,</li> <li>tst no 14: 142004 missing words,</li> <li>tst no 15: 141425 missing words,</li> <li>tst no 16: 142197 missing words,</li> <li>tst no 17: 141595 missing words,</li> <li>tst no 18: 141883 missing words,</li> <li>tst no 19: 141233 missing words,</li> <li>tst no 19: 141233 missing words,</li> <li>tst no 10: 141233 mis</li></ul>				-	-	-	
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tst no 19: 141233 missing words, -1.58 sigmas from mean, p-value= .05703		141883 missing	words,06	sigmas :	from mean, j	p-value=	.47547
tst no 20: 141844 missing words,15 sigmas from mean, p-value= .43934			words, -1.58	8 sigmas :	from mean, j	p-value=	.05703
	tst no 20: 1	141844 missing	words,15	sigmas	from mean, j	p-value=	.43934

Patent rights are pending in this technology. No license to or transfer of any intellectual property rights are granted expressly or by implication, estoppel, or otherwise by this document. This document is furnished with no confidentiality requirement.

:: The tests OPSO, OQSO and DNA : : :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: :: The OPSO test considers 2-letter words from an alphabet of :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: : : :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: :: integers being tested. It considers 10-letter words, so that :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block4.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p)

							mw	Z	р
OPSO fo	r block4.rng	using	bits	23	to	32	142673	2.633	.9958
OPSO fo	r block4.rng	using	bits	22	to	31	142082	.595	.7242
OPSO fo	r block4.rng	using	bits	21	to	30	141722	646	.2592
OPSO fo	r block4.rng	using	bits	20	to	29	141657	870	.1921
OPSO fo	r block4.rng	using	bits	19	to	28	142015	.364	.6422
OPSO fo	r block4.rng	using	bits	18	to	27	141571	-1.167	.1217
OPSO fo	r block4.rng	using	bits	17	to	26	141972	.216	.5855
OPSO fo	r block4.rng	using	bits	16	to	25	142409	1.723	.9576
OPSO fo	r block4.rng	using	bits	15	to	24	142098	.651	.7423
OPSO fo	r block4.rng	using	bits	14	to	23	142127	.751	.7736
OPSO fo	r block4.rng	using	bits	13	to	22	142402	1.699	.9553
OPSO fo	r block4.rng	using	bits	12	to	21	141893	056	.4775
OPSO fo	r block4.rng	using	bits	11	to	20	142448	1.857	.9684
OPSO fo	r block4.rng	using	bits	10	to	19	142081	.592	.7231
OPSO fo	r block4.rng	using	bits	9	to	18	141887	077	.4693

OPSO for block4.rng	using bits	8 to	17	141902025	.4899
OPSO for block4.rng	using bits	7 to		141934 .085	.5339
OPSO for block4.rng	using bits	6 to		141341 -1.960	.0250
OPSO for block4.rng	using bits	5 to		141717663	.2536
OPSO for block4.rng	using bits	4 to		142229 1.102	.8648
	5				
OPSO for block4.rng	using bits	3 to		141961 .178	.5707
OPSO for block4.rng	using bits	2 to		142307 1.371	.9149
OPSO for block4.rng	using bits	1 to	10	141882094	.4625
OQSO test for generator bl	.ock4.rng				
Output: No. missing words	(mw), equiv r	normal	variate	(z), p-value (p	)
	-			mw z	р
OQSO for block4.rng	using bits 2	28 to	32	141386 -1.774	.0380
OQSO for block4.rng	using bits 2			141746554	.2899
OQSO for block4.rng	using bits 2			142032 .416	.6612
OQSO for block4.rng	using bits 2			141764493	.3111
OQSO for block4.rng	using bits 2			141991 .277	.6091
OQSO for block4.rng	using bits 2			141616994	.1600
OQSO for block4.rng	using bits 2	22 to	26	141331 -1.960	.0250
OQSO for block4.rng	using bits 2	21 to	25	141591 -1.079	.1403
OQSO for block4.rng	using bits 2	20 to	24	141644899	.1842
OQSO for block4.rng	using bits 1			142023 .385	.6500
OQSO for block4.rng	using bits 1			141337 -1.940	.0262
OQSO for block4.rng	using bits 1			142304 1.338	.9095
OQSO for block4.rng	using bits 1			142022 .382	.6487
OQSO for block4.rng	using bits 1			141933 .080	.5320
OQSO for block4.rng	using bits 1			141810337	.3682
OQSO for block4.rng	using bits 1			142403 1.673	.9529
OQSO for block4.rng	using bits 1	12 to	16	142252 1.162	.8773
OQSO for block4.rng	using bits 1	11 to	15	142016 .362	.6412
OQSO for block4.rng	using bits 1	10 to	14	141487 -1.432	.0761
OQSO for block4.rng	using bits	9 to	13	141672805	.2106
OQSO for block4.rng	using bits	8 to		141431 -1.621	.0525
OQSO for block4.rng	using bits	7 to		141849205	.4190
OQSO for block4.rng	using bits	, to 6 to		141523 -1.310	.0952
OQSO for block4.rng	using bits	5 to	9	142546 2.158	.9845
OQSO for block4.rng	using bits	4 to	8	142079 .575	.7174
OQSO for block4.rng	using bits	3 to	7	141920 .036	.5144
OQSO for block4.rng	using bits	2 to	б	142015 .358	.6399
OQSO for block4.rng	using bits	1 to	5	141813327	.3720
DNA test for generator bl	.ock4.rng				
Output: No. missing words	(mw), equiv r	normal	variate	(z), p-value (p	)
	· · · · <b>-</b>			mw z	р
DNA for block4.rng	using bits 3	31 t.o	32	142118 .616	.7309
DNA for block4.rng	using bits 3			142456 1.613	.9466
DNA for block4.rng	using bits 2			141915 .017	.5067
	5				
DNA for block4.rng	using bits 2			141888063	.4749
DNA for block4.rng	using bits 2			142278 1.088	.8616
DNA for block4.rng	using bits 2			142398 1.442	.9253
DNA for block4.rng	using bits 2			141698623	.2665
DNA for block4.rng	using bits 2	24 to	25	142306 1.170	.8790
DNA for block4.rng	using bits 2	23 to	24	142123 .630	.7358
DNA for block4.rng	using bits 2			142329 1.238	.8921
DNA for block4.rng	using bits 2			141377 -1.570	.0582
DNA for block4.rng	using bits 2			142162 .745	.7720
	~				
DNA for block4.rng	using bits 1			141743491	.3118
DNA for block4.rng	using bits 1			141916 .020	.5079
DNA for block4.rng	using bits 1	⊥/ to	Tβ	142169 .766	.7782

DNA for 1	block4.rng	using	bits	16	to	17	141629	827	.2041
DNA for ]	block4.rng	using	bits	15	to	16	141782	376	.3536
DNA for ]	block4.rng	using	bits	14	to	15	142153	.719	.7639
DNA for ]	block4.rng	using	bits	13	to	14	142201	.860	.8052
DNA for ]	block4.rng	using	bits	12	to	13	141920	.031	.5126
DNA for ]	block4.rng	using	bits	11	to	12	142227	.937	.8256
DNA for 1	block4.rng	using	bits	10	to	11	142588	2.002	.9774
DNA for ]	block4.rng	using	bits	9	to	10	141927	.052	.5208
DNA for ]	block4.rng	using	bits	8	to	9	141905	013	.4949
DNA for 1	block4.rng	using	bits	7	to	8	141491	-1.234	.1086
DNA for ]	block4.rng	using	bits	6	to	7	142005	.282	.6111
DNA for ]	block4.rng	using	bits	5	to	б	142228	.940	.8264
DNA for 1	block4.rng	using	bits	4	to	5	141654	753	.2257
DNA for ]	block4.rng	using	bits	3	to	4	142163	.748	.7729
DNA for 1	block4.rng	using	bits	2	to	3	142142	.686	.7538
DNA for 1	block4.rng	using	bits	1	to	2	142035	.371	.6446

This is the COUNT-THE-1's TEST on a stream of bytes. :: :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5<sup>5</sup> :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: :: :: for each word. The quadratic form in the weak inverse of :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block4.rng Chi-square with 5^5-5^4=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block4.rng 2448.79 -.724 .234453 byte stream for block4.rng 2432.88 -.949 .171271 

:: This is the COUNT-THE-1's TEST for specific bytes. :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left-:: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter ::

:: hitting five keys with with various probabilities:: 37,56,70,:: :: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2475.21 -.351 .362958 bits 2 to 9 2482.52 -.247 .402366 bits 3 to 10 2638.34 1.956 .974796 bits 4 to 11 2528.06 .397 .654256 bits 5 to 12 .354010 2473.52 -.375 2475.82 -.342 .366174 bits 6 to 13 -.109 bits 7 to 14 2492.27 .456466 bits 8 to 15 .030 2502.12 .511973 bits 9 to 16 2561.13 .865 .806365 bits 10 to 17 2496.56 -.049 .480585 bits 11 to 18 2471.19 -.408 .341818 bits 12 to 19 2496.23 -.053 .478756 bits 13 to 20 2499.48 -.007 .497068 bits 14 to 21 2439.40 -.857 .195723 bits 15 to 22 2468.71 -.443 .329055 bits 16 to 23 2469.37 -.433 .332431 bits 17 to 24 2517.06 .241 .595329 .792 bits 18 to 25 2556.00 .785821 bits 19 to 26 2502.17 .031 .512268 bits 20 to 27 2439.86 -.851 .197505 bits 21 to 28 2523.74 .336 .631465 bits 22 to 29 2594.99 1.343 .910422 .316674 bits 23 to 30 2466.27 -.477 .575654 bits 24 to 31 2513.49 .191 .281 bits 25 to 32 2519.88 .610725

:: THIS IS A PARKING LOT TEST :: :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: :: curve that should be similar to those provided by a perfect :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st- :: :: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block4.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 z-score: -.685 p-value: .246694 Successes: 3508 Successes: 3536 z-score: .594 p-value: .723613 z-score: .411 p-value: .659449 Successes: 3532 Successes: 3540 z-score: .776 p-value: .781201 Successes: 3520 z-score: -.137 p-value: .445521 Successes: 3511 z-score: -.548 p-value: .291865 Successes: 3470 z-score: -2.420 p-value: .007758 Successes: 3549 z-score: 1.187 p-value: .882429 Successes: 3528 z-score: .228 p-value: .590298 Successes: 3518 z-score: -.228 p-value: .409702 square size avg. no. parked sample sigma 100. 3521.200 21.018 KSTEST for the above 10: p= .108721

:: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the  $(n^2-n)/2$  pairs of points. If the points are truly inde- :: :: pendent uniform, then d^2, the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995 . Thus 1-exp(-d^2/.995) should be uniform on [0,1) and :: :: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 :: :: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. :: 

This is the MINIMUM DISTANCE test

for random integers in the file block4.rng

± •	or ranaom	THEEJETD	TH CHC TIT
Sample no.	d^2	avg	equiv uni
5	1.6292	1.3676	.805505
10	3.8078	1.8245	.978223
15	.6531	1.3694	.481281
20	.3982	1.1326	.329785
25	.7857	1.0973	.546014
30	3.3993	1.2014	.967169
35	.3990	1.0973	.330366
40	.1931	1.1490	.176409
45	.8038	1.0844	.554174
50	.0588	1.0096	.057397
55	1.4598	.9823	.769418
60	.5192	.9575	.406552
65	.9622	1.0115	.619781
70	3.6999	1.0399	.975729
75	2.1256	1.0136	.881904
80	.0657	.9929	.063905

1.9800 1.0975 95 .863294 100 .1482 1.0511 .138368 MINIMUM DISTANCE TEST for block4.rng Result of KS test on 20 transformed mindist^2's: p-value= .524298 :: THE 3DSPHERES TEST :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block4.rng r^3= 22.859 sample no: 1 p-value= .53325 sample no: 2 r^3= 2.518 p-value= .08052 sample no: 3 r^3= 2.226 p-value= .07150 sample no: 4 r^3= 19.656 p-value= .48067 sample no: 5 r^3= p-value= .22274 7.559 1.100 sample no: 6 r^3= p-value= .03600 sample no: 7 6.911 r^3= p-value= .20575 sample no: 8 r^3= 28.342 p-value= .61122 sample no: 9 r^3= 22.012 p-value= .51988 sample no: 10 r^3= 10.309 p-value= .29081 r^3= 68.051 sample no: 11 p-value= .89652 p-value= .42305 sample no: 12 r^3= 16.500 sample no: 13 r^3= 31.768 p-value= .65317 sample no: 14 r^3= 36.106 p-value= .69987 sample no: 15 r^3= 20.741 p-value= .49911 sample no: 16 r^3= 2.906 p-value= .09231 r^3= 43.456 sample no: 17 p-value= .76509 sample no: 18 r^3= 16.930 p-value= .43126 sample no: 19 r^3= 6.718 p-value= .20063 sample no: 20 r^3= 55.060 p-value= .84044 A KS test is applied to those 20 p-values. \_\_\_\_\_ 3DSPHERES test for file block4.rng p-value= .564876 :: This is the SQEEZE test :: Random integers are floated to get uniforms on [0,1). Start- :: :: ing with  $k=2^{31}=2147483647$ , the test finds j, the number of :: :: iterations necessary to reduce k to 1, using the reduction :: :: :: k=ceiling(k\*U), with U provided by floating integers from :: :: the file being tested. Such j's are found 100,000 times, :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$  :: :: are used to provide a chi-square test for cell frequencies. ::

.286383

.943640

1.0291

1.0488

.3357

2.8616

85

90

RESULTS OF SQUEEZE TEST FOR block4.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: .8 .8 -.1 .6 -1.2 .3 .2 1.4 -.8 -.1 -.2 -1.5 .8 -.2 -1.3 -.1 .1 -1.6 1.1 -.5 1.7 .5 .3 1.2 .9 -.8 -.7 -1.4 1.6 .0 1.4 .9 1.5 -1.0 -1.0 .1 -1.0 -1.2 2.6 .1 .0 .0 1.8 Chi-square with 42 degrees of freedom: 45.074 z-score= .335 p-value= .655516

The OVERLAPPING SUMS test :: :: :: Integers are floated to get a sequence  $U(1), U(2), \ldots$  of uni-:: :: form [0,1) variables. Then overlapping sums, :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: p-value .303030 Test no. 1 Test no. 2 p-value .396281 Test no. 3 p-value .628466 Test no. 4 p-value .047756 Test no. 5 p-value .284294 p-value .225097 Test no. 6 Test no. 7 p-value .904057 Test no. 8 p-value .596582 Test no. 9 p-value .050642 Test no. 10 p-value .713517 Results of the OSUM test for block4.rng KSTEST on the above 10 p-values: .390328 

:: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: :: weak inverses of the covariance matrices. Runs are counted :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. ::

The RUNS test for file block4.rng Up and down runs in a sample of 10000

					_
		un test for			
		test for 1			
run		test for 1	-		
201		un test for			
		test for 1 test for 1			
r un	S down, KS	iest for i	0 p.s000	030	
\$					
:::					
:: This is the CRAPS TEST. It plays 200,000 games of craps, finds::					
:: the number of wins and the number of throws necessary to end $$ ::					
:: each game. The number of wins should be (very close to) a ::					
:: normal with mean 200000p and variance 200000p(1-p), with ::					
:: p=244/495. Throws necessary to complete the game can vary ::					
<pre>:: from 1 to infinity, but counts for all&gt;21 are lumped with 21. :: :: A chi-square test is made on the noof-throws cell counts. ::</pre>					
:: Each 32-bit integer from the test file provides the value for ::					
:: the throw of a die, by floating to [0,1), multiplying by 6 ::					
:: and taking 1 plus the integer part of the result. ::					
Results of craps test for block4.rng					
No. of	wins: Ob	served Expe	cted		
			98616	98585.86	-
77	ulu of mbo			score= .	.135 pvalue= .55362
Analysis of Throws-per-Game: Chisq= 20.73 for 20 degrees of freedom, p= .58697					
CIIISq-		ows Observe			Sum
	1111	1 66428	66666.7	.854	.854
		2 37432		1.313	2.167
		3 27005	26954.7	.094	2.261
		4 19625	19313.5	5.025	7.286
		5 14047		2.762	10.048
		6 9955		.013	10.061
		7 7117		.110	10.171
		8 5119 9 3638	5139.1 3699.9	.078	10.249
	1	9 3638 .0 2672		1.034 .012	11.284 11.296
		1 1968	1923.3	1.038	12.333
		.2 1357	1388.7	.725	13.059
	1	.3 992	1003.7	.137	13.196
	1	.4 765	726.1	2.080	15.275
	1	.5 512	525.8	.364	15.639
		.6 379	381.2	.012	15.651
		.7 273	276.5	.045	15.697
		.8 173	200.8	3.856	19.553
		.9 158	146.0	.989	20.542
		105 105 105 105 105 105 105 105 105 105	106.2 287.1	.014 .176	20.556 20.732
SUMMARY FOR block4.rng					
p-value for no. of wins: .553618					
		value for t			
	-		-		

Results of DIEHARD battery of tests sent to file report4.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p > .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean m<sup>3</sup>/(4n). Experience shows n :: :: must be quite large, say n>=2^18, for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: :: provide a sample for a KSTEST. BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block5.rng For a sample of size 500: mean block5.rng using bits 1 to 24 2.166 duplicate number number spacings observed expected 0 57. 67.668 1 138. 135.335 2 121. 135.335 94. 3 90.224 4 51. 45.112 5 25. 18.045 6 to INF 14. 8.282 Chisquare with 6 d.o.f. = 10.81 p-value= .905516 For a sample of size 500: mean block5.rng using bits 2 to 25 2.086 duplicate number number

observed spacings expected 67.668 0 66. 135.335 1 137. 2 120. 135.335 3 92. 90.224 4 48. 45.112 5 26. 18.045 11. 6 to INF 8.282 Chisquare with 6 d.o.f. = 6.42 p-value= .622026 For a sample of size 500: mean block5.rng using bits 3 to 26 2.064 duplicate number number spacings observed expected 0 58. 67.668 1 141. 135.335 2 117. 135.335 3 114. 90.224 4 45. 45.112 5 18.045 17. 6 to INF 8.282 8. Chisquare with 6 d.o.f. =10.44 p-value= .892641 For a sample of size 500: mean block5.rng using bits 4 to 27 1.978 duplicate number number expected spacings observed 0 68. 67.668 1 147. 135.335 2 131. 135.335 3 82. 90.224 4 44. 45.112 5 18. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =2.28 p-value= .107709 For a sample of size 500: mean using bits 5 to 28 block5.rng 1.912 number duplicate number spacings observed expected 73. 67.668 0 1 140. 135.335 2 137. 135.335 3 80. 90.224 4 54. 45.112 5 18.045 11. 5. 8.282 6 to INF Chisquare with 6 d.o.f. = 7.56 p-value= .728012 For a sample of size 500: mean block5.rng using bits 6 to 29 1.910 duplicate number number spacings observed expected 0 86. 67.668 1 125. 135.335 2 130. 135.335 3 99. 90.224

4 36. 45.112 5 18.045 18. 6 to INF 8.282 6. Chisquare with 6 d.o.f. =9.29 p-value= .842040 For a sample of size 500: mean block5.rng using bits 7 to 30 2.026 duplicate number number spacings observed expected 0 66. 67.668 1 135.335 136. 2 145. 135.335 3 80. 90.224 4 43. 45.112 5 11. 18.045 6 to INF 19. 8.282 Chisquare with 6 d.o.f. = 18.61 p-value= .995131 For a sample of size 500: mean using bits 8 to 31 1.992 block5.rng duplicate number number spacings observed expected 0 57. 67.668 1 143. 135.335 2 142. 135.335 3 94. 90.224 4 36. 45.112 5 25. 18.045 6 to INF 3. 8.282 .894594 Chisquare with 6 d.o.f. = 10.49 p-value= For a sample of size 500: mean block5.rng using bits 9 to 32 1.996 duplicate number number spacings observed expected 0 56. 67.668 1 146. 135.335 2 137. 135.335 3 90.224 95. 4 45. 45.112 5 18.045 14. 6 to INF 7. 8.282 Chisquare with 6 d.o.f. = 4.23 p-value= .354530 The 9 p-values were .905516 .622026 .892641 .107709 .728012 .995131 .842040 .894594 .354530

A KSTEST for the 9 p-values yields

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:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers ::

.969266

:: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block5.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=110.661; p-value= .801085 OPERM5 test for file block5.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=105.649; p-value= .694874 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block5.rng Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer observed expected (o-e)^2/e sum rank 1.124 28 196 211.4 1.124385 .187059 29 5165 5134.0 1.311 .327972 30 23016 23103.0 1.639 11623 11551.5 .442258 31 2.082 chisquare= 2.082 for 3 d. of f.; p-value= .518910 \_\_\_\_\_ :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29.: : Binary rank test for block5.rng Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer rank observed expected  $(o-e)^2/e$  sum 29 204 211.4 .260276 .260 30 5195 5134.0 .724531 .985 .047003 31 23136 23103.0 1.032 11551.5 32 11465 .648094 1.680 chisquare= 1.680 for 3 d. of f.; p-value= .456058 

:: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: :: random matrices, and a chi-square test is performed on :: counts for ranks 6,5 and <=4. :: Binary Rank Test for block5.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 962 944.3 .332 .332 r =5 21694 21743.9 .115 .446 r =б 77344 77311.8 .013 .460 p=1-exp(-SUM/2)=.20533Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 953 944.3 .080 .080 1.935 r =5 21949 21743.9 2.015 77098 77311.8 2.606 r =6 .591 p=1-exp(-SUM/2)=.72829Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 3 to 10  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM 1.105 944.3 r<=4 912 1.105 r =5 21553 21743.9 1.676 2.781 r =6 77535 77311.8 .644 3.425 p=1-exp(-SUM/2)=.81961Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .575 r<=4 921 944.3 .575 r =5 21655 21743.9 .363 .938 .163 r =б 77424 77311.8 1.101 p=1-exp(-SUM/2)=.42341Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 894 944.3 2.679 2.679 r =5 21551 21743.9 1.711 4.391 77555 77311.8 .765 5.156 r =6 p=1-exp(-SUM/2)=.92407Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .317 .317 927 r<=4 .785 21643 21743.9 .468 r =5 77430 r =6 77311.8 .181 .966

p=1-exp(-SUM/2)=.38304Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .355 926 944.3 .355 r<=4 .837 21609 21743.9 r =5 1.192 77465 77311.8 .304 1.495 r =б p=1-exp(-SUM/2)=.52649Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 976 944.3 1.064 1.064 r<=4 r =5 21939 21743.9 1.751 2.815 r =б 77085 77311.8 .665 3.480 p=1-exp(-SUM/2)=.82448Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .248 r<=4 929 944.3 .248 r =5 21718 21743.9 .031 .279 r =6 77353 77311.8 .301 .022 p=1-exp(-SUM/2)=.13961Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 943 944.3 .002 .002 21843 21743.9 .452 .453 r =5 r =б 77214 77311.8 .124 .577 p=1-exp(-SUM/2)=.25068Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 955 944.3 .121 r<=4 .121 21743.9 r =5 21837 .399 .520 r =б 77208 77311.8 .139 .659 p=1-exp(-SUM/2)=.28079Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 918 944.3 .733 .733 r =5 21864 21743.9 .663 1.396 77218 77311.8 .114 1.510 r =6 p=1-exp(-SUM/2)=.52993Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .355 .355 r<=4 926 944.3 21828 21743.9 .325 .680 r =5 77246 77311.8 .056 .736 r =б p=1-exp(-SUM/2)=.30788

Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 14 to 21 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .100 r<=4 954 944.3 .100 r =5 21781 21743.9 .063 .163 .028 .191 77265 77311.8 r =6 p=1-exp(-SUM/2)=.09119Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 985 944.3 1.754 1.754 r =5 21660 21743.9 .324 2.078 r =6 77355 77311.8 .024 2.102 p=1-exp(-SUM/2)=.65040Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 16 to 23 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 958 944.3 .199 .199 .002 .201 r =5 21737 21743.9 r =б 77305 77311.8 .001 .202 p=1-exp(-SUM/2)=.09585Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 941 944.3 .012 .012 r =5 21786 21743.9 .082 .093 .019 77311.8 .113 r =6 77273 p=1-exp(-SUM/2)=.05471Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 18 to 25  $(O-E)^{2}/E$ OBSERVED EXPECTED SUM 3.054 3.054 998 944.3 r<=4 r =5 21837 21743.9 .399 3.452 r =6 77165 77311.8 .279 3.731 p=1-exp(-SUM/2)=.84518Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 19 to 26 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 955 944.3 .121 .121 r =5 21906 21743.9 1.208 1.330 .386 r =б 77139 77311.8 1.716 p=1-exp(-SUM/2)=.57597Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 20 to 27 OBSERVED  $(O-E)^{2}/E$ SUM EXPECTED r<=4 1003 944.3 3.649 3.649 .044 r =5 21713 21743.9 3.693 77311.8 .010 r =6 77284 3.703 p=1-exp(-SUM/2)=.84297Rank of a 6x8 binary matrix,

rows formed from eight bits of the RNG block5.rng b-rank test for bits 21 to 28 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 980 944.3 1.350 1.350 21684 21743.9 r =5 .165 1.515 r =6 77336 77311.8 .008 1.522 p=1-exp(-SUM/2)=.53284Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 22 to 29 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 950 944.3 .034 .034 21782 r =5 21743.9 .067 .101 r =6 77268 77311.8 .025 .126 p=1-exp(-SUM/2)=.06104Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .100 r<=4 954 944.3 .100 .708 r =5 21868 21743.9 .808 .232 77178 77311.8 1.039 r =6 p=1-exp(-SUM/2)=.40532Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 998 944.3 3.054 3.054 r =5 21666 21743.9 .279 3.333 .008 r =6 77336 77311.8 3.340 p=1-exp(-SUM/2)=.81178Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block5.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 937 944.3 .056 .056 r =5 21604 21743.9 .900 .957 77459 77311.8 .280 1.237 r =б p=1-exp(-SUM/2)=.46120TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .205326 .728286 .819611 .423415 .924066 .526494 .139611 .383041 .824479 .250680 .280792 .529926 .307875 .091195 .650404 .095847 .054711 .845183 .575971 .842974 .532837 .061042 .405322 .811781 .461200 brank test summary for block5.rng The KS test for those 25 supposed UNI's yields KS p-value= .147032 \$ :: THE BITSTREAM TEST :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... Consider an alphabet with two "letters", 0 and 1 ::

::

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:: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of  $2^{21+19}$  bits, the :: :: number of missing words j should be (very close to) normally :: :: :: distributed with mean 141,909 and sigma 428. Thus :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. :: THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^{21}$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ tst no 1: 142436 missing words, 1.23 sigmas from mean, p-value= .89075 tst no 2: 141476 missing words, -1.01 sigmas from mean, p-value= .15566 tst no 3: 141961 missing words, .12 sigmas from mean, p-value= .54805 tst no 4: 141534 missing words, -.88 sigmas from mean, p-value= .19026 tst no 5: 141862 missing words, -.11 sigmas from mean, p-value= .45597 tst no 6: 141509 missing words, -.94 sigmas from mean, p-value= .17480 tst no 7: 141257 missing words, -1.52 sigmas from mean, p-value= .06374 -.06 sigmas from mean, p-value= .47734 141885 missing words, tst no 8: tst no 9: 141996 missing words, .20 sigmas from mean, p-value= .58024 tst no 10: 142092 missing words, .43 sigmas from mean, p-value= .66524 142101 missing words, .45 sigmas from mean, p-value= .67286 tst no 11: tst no 12: 141722 missing words, -.44 sigmas from mean, p-value= .33081 tst no 13: 141687 missing words, -.52 sigmas from mean, p-value= .30172 tst no 14: 141570 missing words, -.79 sigmas from mean, p-value= .21394 .91 sigmas from mean, p-value= .81871 tst no 15: 142299 missing words, tst no 16: 142034 missing words, .29 sigmas from mean, p-value= .61458 tst no 17: 141515 missing words, -.92 sigmas from mean, p-value= .17844 .79 sigmas from mean, p-value= .78493 tst no 18: 142247 missing words, tst no 19: 142131 missing words, .52 sigmas from mean, p-value= .69774

141383 missing words,

tst no 20:

The tests OPSO, OQSO and DNA :: :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined ::

-1.23 sigmas from mean, p-value= .10940

:: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: :: integers being tested. It considers 10-letter words, so that :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: :: The standard deviation sigma=339 was determined as for OQSO :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block5.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw Z р OPSO for block5.rng using bits 23 to 32 141425 -1.670 .0475 .5869 OPSO for block5.rng using bits 22 to 31 141973 .220 OPSO for block5.rng using bits 21 to 30 141694 -.743 .2289 OPSO for block5.rng using bits 20 to 29 142800 3.071 .9989 using bits 19 to 28 OPSO for block5.rng 141799 -.380 .3518 using bits 18 to 27 OPSO for block5.rng 142180 .933 .8247 using bits 17 to 26 OPSO for block5.rng 142165 .882 .8110 OPSO for block5.rng using bits 16 to 25 141407 -1.732 .0416 OPSO for block5.rng using bits 15 to 24 141704 -.708 .2395 OPSO for block5.rng using bits 14 to 23 141730 -.618 .2682 OPSO for block5.rng using bits 13 to 22 141674 -.811 .2085 OPSO for block5.rng using bits 12 to 21 142063 .530 .7019 OPSO for block5.rng using bits 11 to 20 141980 .244 .5963 OPSO for block5.rng using bits 10 to 19 142301 1.351 .9116 OPSO for block5.rng using bits 9 to 18 141937 .095 .5380 OPSO for block5.rng using bits 8 to 17 142101 .661 .7457 OPSO for block5.rng using bits 7 to 16 141883 -.091 .4638 OPSO for block5.rng using bits 6 to 15 -.874 141656 .1912 OPSO for block5.rng using bits 5 to 14 142344 1.499 .9330 OPSO for block5.rng using bits 142220 1.071 4 to 13 .8580 .092 OPSO for block5.rng using bits 3 to 12 141936 .5366 OPSO for block5.rng using bits 2 to 11 142459 1.895 .9710 OPSO for block5.rng using bits 1 to 10 141754 -.536 .2961 OQSO test for generator block5.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р .3087 OQSO for block5.rng using bits 28 to 32 141762 -.499 OQSO for block5.rng using bits 27 to 31 141266 -2.181 .0146 using bits 26 to 30 OQSO for block5.rng 141987 .263 .6038 using bits 25 to 29 OQSO for block5.rng 141623 -.971 .1659 OQSO for block5.rng using bits 24 to 28 141769 -.476 .3171 OQSO for block5.rng using bits 23 to 27 141927 .060 .5239 OQSO for block5.rng using bits 22 to 26 141659 -.849 .1981 OQSO for block5.rng using bits 21 to 25 -.198 141851 .4216

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using bits 20 to 24

using bits 19 to 23

using bits 18 to 22

.436

.426

-.632

142038

142035

141723

.6686

.6649

.2638

OQSO for block5.rng

OQSO for block5.rng

OQSO for block5.rng

UQSU	TOT	DIOCKSTING	using					141911	.000	. 5025
		block5.rng	using	bits	16	to	20	142173	.894	.8143
OQSO	for	block5.rng	using	bits	15	to	19	142042	.450	.6735
OQSO	for	block5.rng	using	bits	14	to	18	141463	-1.513	.0651
OQSO	for	block5.rng	using	bits	13	to	17	141657	855	.1962
OQSO	for	block5.rng	using	bits	12	to	16	141961	.175	.5695
0050	for	block5.rng	using	bits	11	to	15	141732	601	
		block5.rng	using					142475		
		block5.rng	using					142126		
		block5.rng	using					141594	-1.069	
		block5.rng	using					141861		
		block5.rng	using						-1.316	
		block5.rng	using				9	141727		
		block5.rng	using					141671		
			using						.199	
		block5.rng							.697	
		block5.rng	using			to		142156		
		or generator blo			т	LU	5	142130	.050	. 1905
		. missing words			200	cmo ]	wariata	()	value (n	)
output	NO NO	. MISSING WOLDS	(IIIW), (	equiv	1101	llia	L Vallate		vaiue (p z	
	for	block wor	uaina	bi+a	21	+ 0	2.2	mw 142050		р .6609
		block5.rng	using using					142050		
		block5.rng								
		block5.rng	using					141991		
		block5.rng	using					141970		
		block5.rng	using					142266		
		block5.rng	using						.560	
		block5.rng	using					142134		
		block5.rng	using						-2.402	
		block5.rng	using					141670		
		block5.rng	using					141910		.5008
		block5.rng	using					141627		
		block5.rng	using					142263		
		block5.rng	using					142075		
		block5.rng	using						.589	
		block5.rng	using					141791		.3635
		block5.rng	using					142231		.8287
		block5.rng	using					141815		
		block5.rng	using					142512		
		block5.rng	using						.265	
		block5.rng	using						.884	.8116
		block5.rng	using					142313		.8831
DNA	for	block5.rng	using	bits	10	to	11	141700	617	.2685
DNA	for	block5.rng	using	bits	9	to	10	141994	.250	.5986
DNA	for	block5.rng	using	bits	8	to	9	142276	1.082	.8603
DNA	for	block5.rng	using	bits	7	to	8	141998	.262	.6032
DNA	for	block5.rng	using	bits	б	to	7	142673	2.253	.9879
		block5.rng	using			to	6	141662	730	.2328
		block5.rng	using			to	5		-1.016	.1549
		block5.rng	using			to	4	142021	.329	.6291
		block5.rng	using			to	3	142102	.568	.7151
		block5.rng	using			to	2	142644		.9849
		2	2			-			-	-
\$\$\$\$\$\$\$	\$\$\$\$\$	\$\$\$\$\$\$\$\$\$\$\$\$\$	\$\$\$\$\$\$	\$\$\$\$\$	\$\$\$\$	\$\$\$\$	\$\$\$\$\$\$\$			

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OQSO for block5.rng using bits 17 to 21 141911 .006 .5023

:: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5^5 :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: The quadratic form in the weak inverse of :: for each word. :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block5.rng

Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value

Results fo	COUNT-THE-1's in	successive byte	s:	
byte stream	for block5.rng	2455.34	632	.263829
byte stream	for block5.rng	2518.12	.256	.601148

:: This is the COUNT-THE-1's TEST for specific bytes. :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left-:: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5<sup>5</sup> possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2428.60 -1.010 .156311 bits 2 to 9 2496.99 -.043 .483023 bits 3 to 10 2600.20 1.417 .921759 2407.92 bits 4 to 11 -1.302 .096415 bits 5 to 12 2553.04 .750 .773386 bits 6 to 13 -1.111 .133179 2421.41 .615 .730671 bits 7 to 14 2543.48 bits 8 to 15 2587.63 1.239 .892382 bits 9 to 16 2577.43 1.095 .863243 bits 10 to 17 2456.08 -.621 .267258

bits	11	to	18	2544.62	.631	.736000
bits	12	to	19	2493.67	090	.464307
bits	13	to	20	2590.92	1.286	.900748
bits	14	to	21	2584.69	1.198	.884494
bits	15	to	22	2550.21	.710	.761156
bits	16	to	23	2503.84	.054	.521670
bits	17	to	24	2444.12	790	.214680
bits	18	to	25	2382.54	-1.661	.048342
bits	19	to	26	2454.89	638	.261760
bits	20	to	27	2492.49	106	.457704
bits	21	to	28	2586.25	1.220	.888716
bits	22	to	29	2434.63	925	.177607
bits	23	to	30	2557.27	.810	.790996
bits	24	to	31	2433.07	947	.171937
bits	25	to	32	2484.27	222	.411975

:: THIS IS A PARKING LOT TEST : : :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: :: random location. (To avoid path problems, consider parking Each attempt leads to either :: :: helicopters rather than cars.) :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect :: :: :: random number generator. Theory for the behavior of such a :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st- :: :: andard normal variable, which, converted to a uniform varia- :: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block5.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 Successes: 3535 z-score: .548 p-value: .708135 Successes: 3517 z-score: -.274 p-value: .392053 Successes: 3511 z-score: -.548 p-value: .291865 Successes: 3514 z-score: -.411 p-value: .340551 Successes: 3539 z-score: .731 p-value: .767486 Successes: 3496 z-score: -1.233 p-value: .108811 Successes: 3538 .685 p-value: .753306 z-score: Successes: 3501 z-score: -1.005 p-value: .157553

square size	avg. no. parked	sample sigma
100.	3524.600	18.408
KSTEST for	the above 10: p=	.102151

Successes: 3555

Successes: 3540

z-score: 1.461 p-value: .928018

z-score: .776 p-value: .781201

:: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the  $(n^2-n)/2$  pairs of points. If the points are truly inde- :: :: pendent uniform, then d^2, the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995. Thus  $1-\exp(-d^2/.995)$  should be uniform on [0,1) and :: :: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 :: :: :: are printed but the KSTEST is based on the full set of 100 :: random choices of 8000 points in the 10000x10000 square. :: This is the MINIMUM DISTANCE test for random integers in the file block5.rng d^2 equiv uni Sample no. avg 1.3199 .4512 .734608 5 10 2.6141 .7362 .927724 15 1.0100 .8543 .637634 1.4339 .8458 20 .763332 .621937 25 .9678 .7770 30 .3322 .9414 .283847 35 .7507 1.0243 .529719 .3455 40 .9960 .293364 45 2.3645 1.0308 .907115 50 .1963 1.0072 .179006 .9980 .4921 .390175 55 60 2.2287 1.0633 .893533 65 1.0328 .8783 .586345 70 1.0267 .7382 .523791 .320257 75 .3841 1.0043 .9695 80 .0208 .020717 .4425 .359021 85 .9502 90 .9851 .6310 .469650 95 .1621 .9486 .150338 100 .2302 .9470 .206559 MINIMUM DISTANCE TEST for block5.rng Result of KS test on 20 transformed mindist^2's: p-value= .503603 :: THE 3DSPHERES TEST :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- ::

Ψ	he SUCDHERES test	t for file block5.rng	
sample no: 1	r^3= 29.363	p-value= .62422	
sample no: 2	$r^{3} = 5.798$	p-value= .17574	
sample no: 3	$r^{3} = 6.909$	p-value= .20571	
sample no: 4	r^3= 26.439	p-value= .58575	
sample no: 5	$r^{3} = 104.293$	p-value= .96908	
sample no: 6	r^3= 5.604	p-value= .17038	
sample no: 7	r^3= 13.282	p-value= .35772	
sample no: 8	r^3= 11.082	p-value= .30885	
sample no: 9	r^3= 25.090	p-value= .56670	
sample no: 10	$r^{3} = 5.849$	p-value= .17713	
sample no: 11	r^3= 4.839	p-value= .14897	
sample no: 12	r^3= 174.992	p-value= .99707	
sample no: 13	r^3= 56.236	p-value= .84658	
sample no: 14	r^3= 63.491	p-value= .87953	
sample no: 15	r^3= 34.281	p-value= .68104	
sample no: 16	r^3= 2.749	p-value= .08756	
sample no: 17	r^3= 146.778	p-value= .99250	
sample no: 18	r^3= .062	p-value= .00206	
sample no: 19	r^3= 7.795	p-value= .22881	
sample no: 20	r^3= 10.852	p-value= .30354	
A KS test is a	pplied to those 2	20 p-values.	
3DSPHERES	test for file b	lock5.rng p-value= .7432	16
\$\$\$\$\$\$\$\$\$\$\$\$	\$\$\$\$\$\$\$\$\$\$\$\$\$	\$	
			:::::
	s is the SQEEZE t		::
		ated to get uniforms on [0,1). Sta:	
		647, the test finds j, the number (	
		reduce k to 1, using the reduction	
		provided by floating integers from	
		Such j's are found 100,000 times,	::
		ber of times j was <=6,7,,47,>=	
		hi-square test for cell frequencies	
			::::::
		EST FOR block5.rng	
	f standardized fi	requency counts	
	/sqrt(exp) )^2		
	king values <=6,5		
	.1 1.4		
.0 .2		2 1.2	
	1 1.2		
	-1.4 .2		
1.1 .5			
	5.0		
.0 -1.0	.1 1.0	1.6 3.0	
1			
		grees of freedom: 43.969	
Z	score= .215 p-	-value= .611928	

## 

:: form [0,1) variables. Then overlapping sums, S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: The S's are virtually normal with a certain covariance mat-:: rix. A linear transformation of the S's converts them to a :: sequence of independent standard normals, which are converted :::: to uniform variables for a KSTEST. The p-values from ten :: KSTESTs are given still another KSTEST. Test no. 1 p-value .634387 .661018 p-value Test no. 2 Test no. 3 p-value .080425 p-value .708840 Test no. 4 Test no. 5 p-value .344657 Test no. .768837 б p-value Test no. 7 p-value .396214 .358390 Test no. 8 p-value Test no. 9 p-value .027324 Test no. 10 p-value .760155 Results of the OSUM test for block5.rng KSTEST on the above 10 p-values: .290043

:: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file block5.rng

Up and down runs in a sample of 10000

Run test for block5.rng runs up; ks test for 10 p's: .741353 runs down; ks test for 10 p's: .212547 Run test for block5.rng : runs up; ks test for 10 p's: .797036 runs down; ks test for 10 p's: .551932

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with : : :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for ::

::

::

::

::

::

::

:: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for block5.rng No. of wins: Observed Expected 98747 98585.86 98747= No. of wins, z-score= .721 pvalue= .76446 Analysis of Throws-per-Game: Chisq= 15.76 for 20 degrees of freedom, p= .26863 Throws Observed Expected Chisq Sum 1 66377 66666.7 1.259 1.259 2 37848 37654.3 .996 2.255 3 27018 26954.7 .149 2.403 4 19237 19313.5 2.706 .303 5 13946 13851.4 .646 3.352 б 9874 9943.5 .486 3.838 7 7168 7145.0 .074 3.912 8 5254 5139.1 2.570 6.482 9 3699.9 6.870 3662 .388 2666.3 10 2654 .057 6.927 11 1918 1923.3 .015 6.941 12 1337 1388.7 1.928 8.869 13 1025 1003.7 .451 9.320 14 715 726.1 .171 9.491 525.8 3.381 15 568 12.872 16 384 381.2 12.894 .021 271 276.5 17 .111 13.004 18 206 200.8 .133 13.138 .172 19 151 146.0 13.310 1.308 20 118 106.2 14.617 1.143 21 269 287.1 15.760 SUMMARY FOR block5.rng p-value for no. of wins: .764458 p-value for throws/game: .268631 

Results of DIEHARD battery of tests sent to file report5.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p> .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". 

:: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings ::  $\colon$  between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block6.rng For a sample of size 500: mean block6.rng using bits 1 to 24 1.916 duplicate number number spacings observed expected 0 71. 67.668 145. 135.335 1 2 143. 135.335 3 74. 90.224 4 41. 45.112 5 18. 18.045 6 to INF 8. 8.282 Chisquare with 6 d.o.f. = 4.59 p-value= .402646 For a sample of size 500: mean block6.rng using bits 2 to 25 2.010 duplicate number number observed expected spacings 0 65. 67.668 1 148. 135.335 2 122. 135.335 3 83. 90.224 4 45.112 56. 5 17. 18.045 9. 6 to INF 8.282 Chisquare with 6 d.o.f. =5.93 p-value= .569311 For a sample of size 500: mean block6.rng using bits 3 to 26 1.926 duplicate number number spacings observed expected 0 74. 67.668 135.335 1 135. 2 137. 135.335 3 92. 90.224 4 37. 45.112 5 18.045 21. 6 to INF 4. 8.282 Chisquare with 6 d.o.f. = 4.81 p-value= .430976

:::::::::			
	For a s	sample of size 500:	mean
ł		using bits 4 to 27	
duplicate	number	number	
spacings	observed	expected	
0	64.	67.668	
1	122.	135.335	
2	139.	135.335	
3	101.	90.224	
4	48.	45.112	
5	20.	18.045	
6 to INF	6.	8.282	
		= 3.92 p-value=	. 31 31 2.9
			101011
	For a s	sample of size 500:	mean
ł	olock6.rng	using bits 5 to 28	1.996
duplicate	number	number	2.000
spacings	observed		
0	63.	67.668	
1	138.	135.335	
2	143.	135.335	
3	86.	90.224	
4	43.	45.112	
5	13.	18.045	
6 to INF	9.	8.282	
Chisquare t	with 6 d o f		021540
	with 6 d.o.f.		.021540
		= 1.17 p-value=	
::::::::	For a s	= 1.17 p-value= ::::::::::::::::::::::::::::::::::::	mean
::::::::	For a s	= 1.17 p-value= ::::::::::::::::::::::::::::::::::::	mean
:::::::: duplicate	For a solock6.rng	= 1.17 p-value= sample of size 500: using bits 6 to 29 number	mean
::::::: duplicate spacings	For a s For a s Fock6.rng number	= 1.17 p-value= sample of size 500: using bits 6 to 29 number expected	mean
:::::::: duplicate	For a s For a s clock6.rng number observed	= 1.17 p-value= semple of size 500: using bits 6 to 29 number expected 67.668	mean
::::::: duplicate spacings 0 1	For a s For a s clock6.rng number observed 64.	= 1.17 p-value= 	mean
::::::: duplicate spacings 0 1 2	For a s For a s block6.rng number observed 64. 139.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean
::::::: duplicate spacings 0 1	For a s For a s olock6.rng number observed 64. 139. 142. 83.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean
::::::: duplicate spacings 0 1 2 3	For a s For a s olock6.rng number observed 64. 139. 142.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean
::::::: duplicate spacings 0 1 2 3 4	For a s For a s olock6.rng number observed 64. 139. 142. 83. 52.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean
::::::: duplicate spacings 0 1 2 3 4 5 6 to INF	For a s For a s plock6.rng number observed 64. 139. 142. 83. 52. 13. 7.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean
::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare v	For a s For a s plock6.rng number observed 64. 139. 142. 83. 52. 13.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978
::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare v	For a s For a s olock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978
<pre>::::::::::::::::::::::::::::::::::::</pre>	For a s For a s olock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. For a s	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978
<pre>::::::::::::::::::::::::::::::::::::</pre>	For a s For a s olock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
<pre>::::::::::::::::::::::::::::::::::::</pre>	For a s For a s olock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::: For a s olock6.rng	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
:::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare v :::::::::	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::: For a solock6.rng number	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare :::::::: duplicate spacings	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::: For a solock6.rng number observed	<pre>= 1.17 p-value=</pre>	mean 1.978 .305071 mean
:::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare ::::::::: duplicate spacings 0	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::::::::::::::::::::::	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
:::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare v ::::::::: duplicate spacings 0 1	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. :::::::::::::: For a solock6.rng number observed 71. 135.	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
:::::::: duplicate spacings 0 1 2 3 4 5 6 to INF Chisquare v ::::::::: duplicate spacings 0 1 2	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::::::::::::::::::::::	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean
<pre>::::::::::::::::::::::::::::::::::::</pre>	For a solock6.rng number observed 64. 139. 142. 83. 52. 13. 7. with 6 d.o.f. ::::::::::::::::::::::::::::::::::	<pre>= 1.17 p-value= ::::::::::::::::::::::::::::::::::::</pre>	mean 1.978 .305071 mean

6 to INF

duplicate

spacings

Patent rights are pending in this technology. No license to or transfer of any intellectual property rights are granted expressly or by implication, estoppel, or otherwise by this document. This document is furnished with no confidentiality requirement.

using bits 8 to 31

7.04 p-value= .683215

mean

2.044

8.282

For a sample of size 500:

number

expected

4.

number

observed

Chisquare with 6 d.o.f. =

block6.rng

0 60. 67.668 1 145. 135.335 2 135.335 134. 3 82. 90.224 4 45. 45.112 5 24. 18.045 10. 8.282 6 to INF Chisquare with 6 d.o.f. =4.64 p-value= .409776 For a sample of size 500: mean using bits 9 to 32 block6.rng 1.918 duplicate number number observed expected spacings 0 53. 67.668 1 157. 135.335 2 148. 135.335 3 90.224 80. 4 47. 45.112 5 18.045 12 6 to INF 3. 8.282 Chisquare with 6 d.o.f. = 14.46 p-value= .975134 The 9 p-values were .569311 .430976 .402646 .021540 .313129 .683215 .305071 .409776 .975134 A KSTEST for the 9 p-values yields .388399

\$

:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: :: are observed, cumulative counts are made of the number of :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block6.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=111.188; p-value= .810631 OPERM5 test for file block6.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=131.353; p-value= .983563 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-::

:: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block6.rng Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 204 211.4 .260276 .260 28 29 4988 5134.0 4.152503 4.413 30 23265 23103.0 1.135297 5.548 31 11543 11551.5 .006291 5.554 chisquare= 5.554 for 3 d. of f.; p-value= .873795 \_\_\_\_\_ :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. :: Binary rank test for block6.rng Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 1.462 211.4 1.462156 29 229 5134.0 .954140 30 5204 2.416 2305023103.0.1218011151711551.5.103184 31 2.538 32 2.641 chisquare= 2.641 for 3 d. of f.; p-value= .601762 \_\_\_\_\_ :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a :: :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: random matrices, and a chi-square test is performed on :: :: counts for ranks 6,5 and <=4. :: ..... Binary Rank Test for block6.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block6.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 947 .008 r<=4 .008 21743.9 21706 r =5 .066 .074 77311.8 r =6 77347 .016 .090 p=1-exp(-SUM/2)=.04391Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block6.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED (O-E)^2/E SUM

	0.55	0.4.40		
r<=4	975	944.3	.998	.998
r =5	21581	21743.9	1.220	2.218
r =6	77444	77311.8	.226	2.444
Deple of a		p(-SUM/2) = .'	/0543	
rows formed fr	x8 binary ma		a blocké rna	
b-rank test for			J DIOCKO.ING	
D-IAIR LEST IC	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	963	944.3	.370	.370
r =5	21804		.166	.536
r =6		77311.8	.080	.617
1 -0		p(-SUM/2) = .2		.01/
Rank of a 6	x8 binary ma			
rows formed fr			G block6.rng	
b-rank test fo			o 21001101111	
	OBSERVED		(O-E)^2/E	SUM
r<=4	901	944.3	1.986	1.986
r =5	21923	21743.9	1.475	3.461
r =6			.239	3.699
	p=1-exp	(-SUM/2) = .8	84271	
Rank of a 6	x8 binary ma			
rows formed fr			G block6.rng	
b-rank test fo	or bits 5 to	b 12		
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	936	944.3	.073	.073
r =5	21862	21743.9	.641	.714
r =6	77202	77311.8	.156	.870
	p=1-exp	p(-SUM/2) = .2	35286	
	x8 binary ma			
rows formed fr			G block6.rng	
b-rank test fo				
	OBSERVED	-	(O-E)^2/E	SUM
r<=4	935	944.3	.092	.092
r =5		21743.9		4.571
r =6		77311.8		5.757
		p(-SUM/2) = .9	94379	
	x8 binary ma			
rows formed fr			J block6.rng	
b-rank test fo	OBSERVED			OTIM
70 c = 1		-	(O-E)^2/E	SUM
r<=4	964 21824	944.3	.411 .295	.411
r =5 r =6	21824 77212	21743.9 77311.8	.1295	.706
T -0		p(-SUM/2) = .2		.035
Pank of a 6	x8 binary ma		54120	
rows formed fr			z block6 rng	
b-rank test fo			J DIOCKO.IIIg	
D TAIM CODE TO	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	944	944.3	.000	.000
r =5	21679	21743.9	.194	.194
r =6	77377	77311.8	.055	.249
_ •		p(-SUM/2) = .2		/
Rank of a 6	x8 binary ma			
rows formed fr			G block6.rng	
b-rank test fo			5	
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	898	944.3	2.270	2.270

<b>-</b>	21.0.01	01740 0	1 5 0	2 4 2 0
r =5 r =6		21743.9 77311.8		$2.420 \\ 2.422$
T =0		p(-SUM/2) = .'		2.422
Rank of a	6x8 binary ma		10200	
	from eight bit		7 block6 rng	
	for bits 10 to		5 Dicentering	
			(O-E)^2/E	SUM
r<=4		944.3		1.350
r =5		21743.9		1.799
r =6	77375		.052	1.851
	p=1-exp	p(-SUM/2) = .6	50368	
Rank of a	6x8 binary ma	atrix,		
rows formed :	from eight bit	ts of the RNG	G block6.rng	
b-rank test :	for bits 11 to			
			(O-E)^2/E	SUM
r<=4	953	944.3	.080	.080
r =5	21719	21743.9	.029	.109
r =6	77328	77311.8	.003	.112
		p(-SUM/2) = .0	05448	
	6x8 binary ma			
	from eight bit		G block6.rng	
b-rank test	for bits 12 to			OTTM
			(O-E)^2/E	SUM 1.554
r<=4 r =5	906	944.3 21743.9	1.554 .089	1.554
	77394			1.730
1 =0		p(-SUM/2) = .5		1.750
Rank of a	6x8 binary ma		57005	
	from eight bit		7 block6.rng	
	for bits 13 to		5 5100no.1ng	
			(O-E)^2/E	SUM
r<=4	917	944.3	.789	.789
r =5	21684	21743.9	.165	.954
r =6		77311.8	.098	1.053
		p(-SUM/2) = .4	40924	
	6x8 binary ma			
	from eight bit		G block6.rng	
b-rank test :	for bits 14 to			
		EXPECTED	(O-E)^2/E	SUM
r<=4	953	944.3	.080	.080
r =5	21869	21743.9	.720	.800
r =6	77178	77311.8	.232	1.031
		p(-SUM/2) = .4	40293	
	6x8 binary ma	•		
	from eight bit for bits 15 to		J DIOCKO.ING	
D-TAIK LEST	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	958	944.3	.199	.199
r = 5		21743.9	.183	.382
r =6	77235		.076	.458
- •		p(-SUM/2) = .2		. 100
Rank of a	6x8 binary ma			
	from eight bit		G block6.rng	
	for bits 16 to		5	
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	919	944.3	.678	.678
r =5	21708	21743.9	.059	.737

r =6	77373	77311.8	.048	.786
		(-SUM/2) = .3	32484	
Rank of a 6 rows formed fr	5x8 binary ma		- block6 rng	
b-rank test for			J DIOCKO.IIIg	
D TAIK LEST IC	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	975	944.3	.998	.998
r =5		21743.9	1.472	2.470
r =6	77460	77311.8		2.754
1 0		(-SUM/2) = .7		2.,51
Rank of a 6	5x8 binary ma		1100	
rows formed fr			B block6.rng	
b-rank test fo				
		EXPECTED	(O-E) <sup>2</sup> /E	SUM
r<=4	969	944.3	.646	.646
r =5	21843	21743.9	.452	1.098
r =6	77188	77311.8	.198	1.296
	p=1-exp	(-SUM/2) = .4	17689	
	5x8 binary ma			
rows formed fr	om eight bit	s of the RNG	G block6.rng	
b-rank test fo	or bits 19 to	26		
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	934	944.3	.112	.112
r =5	21814	21743.9	.226	.338
r =6		77311.8	.046	.385
		(-SUM/2) = .1	L7495	
	5x8 binary ma			
rows formed fr			G block6.rng	
b-rank test fo				
	OBSERVED		(O-E)^2/E	SUM
r<=4	959	944.3	.229	.229
r =5	22044	21743.9	4.142	4.371
r =6	76997		1.282	5.652
		(-SUM/2) = .9	94077	
	5x8 binary ma			
rows formed fr			j blockb.rng	
b-rank test fo	OF DITS 21 to OBSERVED			OTIM
		944.3	(O-E)^2/E	SUM
r<=4 r =5	961 21809	944.3 21743.9	.295 .195	.295 .490
r =6	77230	77311.8	.087	.490
I =0		(-SUM/2) = .2		. 577
Rank of a f	5x8 binary ma		2002	
rows formed fr			- block6 rng	
b-rank test for			bioeko.ing	
D TAIM CEBE IC	OBSERVED		(O-E)^2/E	SUM
r<=4	936	944.3	.073	.073
r = 5		21743.9	.265	.338
r =6		77311.8	.092	.430
_ •		(-SUM/2) = .1		
Rank of a f	5x8 binary ma			
rows formed fr			G block6.rna	
b-rank test fo				
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	942	944.3	.006	.006
r =5	21541	21743.9	1.893	1.899
r =6	77517	77311.8	.545	2.444

p=1-exp(-SUM/2)=.70530Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block6.rng b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 1005 944.3 3.902 3.902 21627 21743.9 .628 4.530 r =5 77311.8 r =б 77368 .041 4.571 p=1-exp(-SUM/2)=.89828Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block6.rng b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 944.3 .135 .135 r<=4 933 r =5 21842 21743.9 .443 .578 r =б 77225 77311.8 .097 .675 p=1-exp(-SUM/2)=.28656TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .705426 .043905 .265344 .842712 .352857 .943789 .341255 .116967 .702058 .603680 .054479 .578851 .409237 .402932 .204726 .324845 .747664 .476888 .174953 .940765 .193301 .705295 .898276 .250522 .286555 brank test summary for block6.rng The KS test for those 25 supposed UNI's yields KS p-value= .199101

:: THE BITSTREAM TEST :: :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... . Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. :: THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^21$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ tst no 1: 142200 missing words, .68 sigmas from mean, p-value= .75148 tst no2:142200 missing words,<br/>tst no.74 sigmas from mean, p-value=.76961tst no3:141968 missing words,<br/>tst no.14 sigmas from mean, p-value=.55452tst no4:142316 missing words,<br/>tst no.95 sigmas from mean, p-value=.82899tst no5:141928 missing words,<br/>tst no.04 sigmas from mean, p-value=.51740tst no6:141668 missing words,<br/>tst no-.56 sigmas from mean, p-value=.28643tst no7:141738 missing words,<br/>tst no-.40 sigmas from mean, p-value=.34447

tst no 8:	142447 missing	words,	1.26	sigmas	from mean,	p-value=	.89549
tst no 9:	141747 missing	words,	38	sigmas	from mean,	p-value=	.35224
tst no 10:	142046 missing	words,	.32	sigmas	from mean,	p-value=	.62526
tst no 11:	141714 missing	words,	46	sigmas	from mean,	p-value=	.32406
tst no 12:	141939 missing	words,	.07	sigmas	from mean,	p-value=	.52764
tst no 13:	141390 missing	words,	-1.21	sigmas	from mean,	p-value=	.11249
tst no 14:	141639 missing	words,	63	sigmas	from mean,	p-value=	.26382
tst no 15:	142330 missing	words,	.98	sigmas	from mean,	p-value=	.83717
tst no 16:	142102 missing	words,	.45	sigmas	from mean,	p-value=	.67371
tst no 17:	141415 missing	words,	-1.15	sigmas	from mean,	p-value=	.12405
tst no 18:	142447 missing	words,	1.26	sigmas	from mean,	p-value=	.89549

-2.57 sigmas from mean, p-value= .00504

-.98 sigmas from mean, p-value= .16246

140808 missing words,

141488 missing words,

tst no 19: tst no 20:

:: The tests OPSO, OQSO and DNA :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten : : :: consecutive bits. It then restarts the file for the next de-:: :: :: signated 10 bits, and so on. :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: :: file, elements of which are assumed 32-bit random integers. :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: :: integers being tested. It considers 10-letter words, so that :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block6.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р 141579 -1.139 OPSO for block6.rng using bits 23 to 32 .1273 OPSO for block6.rng using bits 22 to 31 142028 .409 .6588

OPSO for block6.	rng using	bits	21	to	30	141593	-1.091	.1377
OPSO for block6.	rng using	bits	20	to	29	141516	-1.356	.0875
OPSO for block6.	rng using	bits	19	to	28	142213	1.047	.8525
OPSO for block6.	rng using	bits	18	to	27	142043	.461	.6776
OPSO for block6.	rng using	bits	17	to	26	141772	474	.3179
OPSO for block6.	rng using	bits	16	to	25	141778	453	.3253
OPSO for block6.	rng using	bits	15	to	24	141650	894	.1856
OPSO for block6.	rng using	bits	14	to	23	141720	653	.2569
OPSO for block6.	rng using	bits	13	to	22	141570	-1.170	.1210
OPSO for block6.	rng using	bits	12	to	21	141997	.302	.6188
OPSO for block6.	rng using	bits	11	to	20	142171	.902	.8166
OPSO for block6.	rng using	bits	10	to	19	142010	.347	.6358
OPSO for block6.	rng using	bits	9	to	18	141785	429	.3341
OPSO for block6.		bits	8	to	17	141940	.106	.5421
OPSO for block6.	rng using	bits	7	to	16	141881	098	.4611
OPSO for block6.	rng using	bits	б	to	15	141823	298	.3830
OPSO for block6.	rng using	bits	5	to	14	141844	225	.4109
OPSO for block6.	rng using	bits	4	to	13	142206	1.023	.8468
OPSO for block6.				to			.085	
OPSO for block6.						142040		
OPSO for block6.	5			to		142049		
OQSO test for gener		q						
Output: No. missin			noi	rmal	l variate	(z), p-v	value (p	)
1	5	-				mw	Z	p
OQSO for block6.	rng using	bits	28	to	32	141314	-2.018	.0218
OQSO for block6.						141935	.087	.5347
OQSO for block6.							.524	
OQSO for block6.							1.029	
OQSO for block6.							-2.103	
OQSO for block6.							-1.201	
OQSO for block6.							208	
OQSO for block6.							123	
OQSO for block6.						141990		
OQSO for block6.						142006		
OQSO for block6.						141602	-1.042	
OQSO for block6.	5						2.182	
OQSO for block6.							2.151	
00S0 for block6.							123	
OQSO for block6.							-1.266	.1028
OQSO for block6.						142023		.6500
OQSO for block6.							191	
OQSO for block6.						142224		
OQSO for block6.							.772	
OQSO for block6.							-1.269	
OQSO for block6.							1.840	
OQSO for block6.							-1.232	
OQSO for block6.						141933		
OQSO for block6.							-1.055	
OQSO for block6.							.080	
OQSO for block6.	rnq using						784	
OQSO for block6.	rng using						-1.303	
OQSO for block6.							2.392	
DNA test for gener			Ŧ	ιU	J	142013	2.392	.9910
Output: No. missin			nor	cm o T	variato	(z)	value (n	)
	y words (IIIW),	equiv	1101	llid	r variate	(2), p-v mw		
DNA for block6.	rng using	bi+c	21	+ ~	20		z -1.452	р .0732
DNA for block6.							-1.452 .899	
DINA LUI DIUCKO.	LING UDING	DICO	50	20	J L	_ IZZ_ <del>1</del>		.0100

DNA for block6.rng	using bits 29 to 30	142038 .380	.6479
DNA for block6.rng	using bits 28 to 29	142208 .881	.8109
DNA for block6.rng	using bits 27 to 28	142426 1.524	.9363
DNA for block6.rng	using bits 26 to 27	141704606	.2724
DNA for block6.rng	using bits 25 to 26	142381 1.391	.9179
DNA for block6.rng	using bits 24 to 25	141701615	.2694
DNA for block6.rng	using bits 23 to 24	142267 1.055	.8543
DNA for block6.rng	using bits 22 to 23	142111 .595	.7240
DNA for block6.rng	using bits 21 to 22	141640794	.2135
DNA for block6.rng	using bits 20 to 21	142003 .276	.6088
DNA for block6.rng	using bits 19 to 20	142226 .934	.8249
DNA for block6.rng	using bits 18 to 19	142126 .639	.7386
DNA for block6.rng	using bits 17 to 18	142097 .554	.7101
DNA for block6.rng	using bits 16 to 17	141257 -1.924	.0272
DNA for block6.rng	using bits 15 to 16	142182 .804	.7894
DNA for block6.rng	using bits 14 to 15	141529 -1.122	.1309
DNA for block6.rng	using bits 13 to 14	141295 -1.812	.0350
DNA for block6.rng	using bits 12 to 13	141449 -1.358	.0872
DNA for block6.rng	using bits 11 to 12	142583 1.987	.9766
DNA for block6.rng	using bits 10 to 11	142362 1.335	.9091
DNA for block6.rng	using bits 9 to 10	142347 1.291	.9017
DNA for block6.rng	using bits 8 to 9	141719561	.2872
DNA for block6.rng	using bits 7 to 8	142138 .675	.7500
DNA for block6.rng	using bits 6 to 7	141281 -1.853	.0319
DNA for block6.rng	using bits 5 to 6	142163 .748	.7729
DNA for block6.rng	using bits 4 to 5	141434 -1.402	.0804
DNA for block6.rng	using bits 3 to 4	141758446	.3277
DNA for block6.rng	using bits 2 to 3	141940 .090	.5360
DNA for block6.rng	using bits 1 to 2	142016 .315	.6235

### 

byte stream for block6.rng

:: This is the COUNT-THE-1's TEST on a stream of bytes. :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5<sup>5</sup> :: :: possible 5-letter words, and from a string of 256,000 (over- :: :: lapping) 5-letter words, counts are made on the frequencies :: :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block6.rng Chi-square with 5^5-5^4=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block6.rng .542 2538.34 .706167

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2490.07

-.140

.444178

This is the COUNT-THE-1's TEST for specific bytes. :: :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left-:: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2419.56 -1.138 .127633 bits 2 to 9 2512.94 .183 .572592 bits 3 to 10 2502.42 .034 .513622 bits 4 to 11 2358.81 -1.997 .022928 .749 .773104 bits 5 to 12 2552.97 bits 6 to 13 2418.89 -1.147.125672 bits 7 to 14 2541.83 .592 .722935 bits 8 to 15 2421.25 -1.114.132696 bits 9 to 16 2504.60 .065 .525929 bits 10 to 17 2361.67 -1.956 .025216 bits 11 to 18 2636.87 1.936 .973543 .770 bits 12 to 19 2554.43 .779274 bits 13 to 20 2490.80 -.130 .448249 bits 14 to 21 -1.4522397.33 .073247 bits 15 to 22 2590.43 1.279 .899540 bits 16 to 23 2452.54 -.671 .251056 bits 17 to 24 2563.16 .893 .814139 bits 18 to 25 2469.13 -.437 .331214 bits 19 to 26 2568.96 .975 .835283 bits 20 to 27 2468.41 -.447 .327504 bits 21 to 28 2480.98 -.269 .393993 bits 22 to 29 2613.70 1.608 .946079 bits 23 to 30 2635.35 1.914 .972201 bits 24 to 31 2480.06 -.282 .388962 bits 25 to 32 2574.81 1.058 .854972

		:::
::	THIS IS A PARKING LOT TEST	::
:: In a square	of side 100, randomly "park" a cara circle of	::
:: radius 1.	Then try to park a 2nd, a 3rd, and so on, each	::

:: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: :: curve that should be similar to those provided by a perfect :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st- :: :: andard normal variable, which, converted to a uniform varia- :: :: ble, provides input to a KSTEST based on a sample of 10. :: 

CDPARK:	result	of	ten	tests	on	file	block6.rng

Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9

Successes:	3520	z-score:	137	p-value:	.445521
Successes:	3543	z-score:	.913	p-value:	.819442
Successes:	3509	z-score:	639	p-value:	.261324
Successes:	3522	z-score:	046	p-value:	.481790
Successes:	3501	z-score:	-1.005	p-value:	.157553
Successes:	3517	z-score:	274	p-value:	.392053
Successes:	3493	z-score:	-1.370	p-value:	.085365
Successes:	3527	z-score:	.183	p-value:	.572463
Successes:	3515	z-score:	365	p-value:	.357445
Successes:	3491	z-score:	-1.461	p-value:	.071982

square size avg. no. parked sample sigma 100. 3513.800 15.098 KSTEST for the above 10: p= .780431

:: THE MINIMUM DISTANCE TEST :: :: It does this 100 times:: choose n=8000 random points in a :: :: square of side 10000. Find d, the minimum distance between :: :: the (n^2-n)/2 pairs of points. If the points are truly inde- :: :: pendent uniform, then d^2, the square of the minimum distance :: :: should be (very close to) exponentially distributed with mean :: :: .995 . Thus 1-exp(-d^2/.995) should be uniform on [0,1) and :::: a KSTEST on the resulting 100 values serves as a test of uni- :: :: formity for random points in the square. Test numbers=0 mod 5 :: :: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. :: This is the MINIMUM DISTANCE test for random integers in the file block6.rng

avg equiv	uni
2477 .568	938
7447 .231	089
8470 .276	503
	avg equiv 2477 .5689 7447 .2310 8470 .2760

20	.3293	.7405	.281760	
25	.8094	.7324	.556671	
30	1.5835	.8458	.796370	
35	1.6999	.8546	.818850	
40	1.6143	.8828	.802586	
45	.2280	.8792	.204765	
50	.7023	.9437	.506296	
55	.1686	.8916	.155858	
60	.2770	.9191	.243006	
65	.2577	.9762	.228178	
70	.0445	.9167	.043757	
75	.4939	.9254	.391254	
80	1.2292	.9250	.709286	
85	.9900	.9105	.630264	
90	.1601	.9159	.148615	
95	.0992	.8776	.094906	
100	2.6976	.9206	.933539	
MINIMUM	DISTANCE T	EST for blo	ck6.rng	
_	7			-

Result of KS test on 20 transformed mindist<sup>2</sup>'s: p-value= .233572

#### 

:: THE 3DSPHERES TEST :: :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. :: Thus :: the radius cubed is exponential with mean 30. (The mean is :: :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. : : The 3DSPHERES test for file block6.rng r^3= p-value= .83627 sample no: 54.287 1 r^3= sample no: 2 30.124 p-value= .63364 r^3= p-value= .61644 sample no: 3 28.748 r^3= sample no: 4 4.407 p-value= .13663 r^3= p-value= .13189 sample no: 5 4.243 sample no: б r^3= 26.822 p-value= .59101 sample no: 7 r^3= 46.478 p-value= .78759 sample no: 8 r^3= 12.604 p-value= .34303 sample no: 9 r^3= 34.737 p-value= .68586 sample no: 10 r^3= 8.896 p-value= .25661 r^3= sample no: 11 .308 p-value= .01022 r^3= sample no: 12 37.168 p-value= .71031 sample no: 13 r^3= p-value= .18893 6.282 sample no: 14 r^3= p-value= .32766 11.910 sample no: 15 r^3= p-value= .77248 44.415 sample no: 16 r^3= 34.185 p-value= .68002 sample no: 17 r^3= 38.573 p-value= .72356 sample no: 18 r^3= 45.426 p-value= .78002 8.581 sample no: 19 r^3= p-value= .24876 r^3= 5.272 sample no: 20 p-value= .16117

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A KS test is applied to those 20 p-values.

\_\_\_\_\_ 3DSPHERES test for file block6.rng p-value= .461227 :: This is the SQEEZE test :: :: Random integers are floated to get uniforms on [0,1). Start- :: :: ing with  $k=2^{31}=2147483647$ , the test finds j, the number of :: :: iterations necessary to reduce k to 1, using the reduction :: :: k=ceiling(k\*U), with U provided by floating integers from :: :: the file being tested. Such j's are found 100,000 times, :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$  :: :: are used to provide a chi-square test for cell frequencies. :: RESULTS OF SQUEEZE TEST FOR block6.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: -1.5 -.7 -.1 -1.6 2.6 .3 .0 -.8 -.8 .8 .5 1.1 .9 .7 1.4 .2 -1.6 -.3 2.8 -.3 -.5 -1.1 -.б .4 -.4 .0 -.9 -1.5 -.2 -.1 .5 2.7 -1.1 -.3 -.1 .8 .1 .4 .9 2.0 .5 -1.3 -.1 Chi-square with 42 degrees of freedom: 50.856 z-score= .966 p-value= .835971

#### 

:: The OVERLAPPING SUMS test :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: p-value .467835 Test no. 1 Test no. 2 p-value .684353 Test no. 3 p-value .765742 p-value .667671 p-value .748993 p-value .676145 Test no. 4 Test no. 5 Test no. 6 p-value .356360 Test no. 7 Test no. 8 p-value .606335 p-value Test no. 9 .297356 Test no. 10 p-value .757876 Results of the OSUM test for block6.rng KSTEST on the above 10 p-values: .870741

:: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float-:: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: 

:

The RUNS test for file block6.rng Up and down runs in a sample of 10000

Run test for block6.rng runs up; ks test for 10 p's: .095465 runs down; ks test for 10 p's: .728443 Run test for block6.rng runs up; ks test for 10 p's: .408193 runs down; ks test for 10 p's: .164630

#### 

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: :: normal with mean 200000p and variance 200000p(1-p), with :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: 

Results of craps test for block6.rng

No. of wins: Observed Expected

#### 98798 98585.86

98798= No. of wins, z-score= .949 pvalue= .82864

Analysis of Throws-per-Game:

Chisq= 20.19 for 20 degrees of freedom, p= .55382 Throws Observed Expected Chisq Sum 1 66478 66666.7 .534 .534 2 37973 37654.3 2.697 3.231

_				
2	37973	37654.3	2.697	3.231
3	26936	26954.7	.013	3.244
4	19347	19313.5	.058	3.302
5	13745	13851.4	.818	4.120
6	9827	9943.5	1.366	5.486
7	7157	7145.0	.020	5.506
8	5136	5139.1	.002	5.508
9	3695	3699.9	.006	5.514
10	2603	2666.3	1.503	7.017
11	1921	1923.3	.003	7.020
12	1456	1388.7	3.258	10.277

13	1036	1003.7	1.038	11.316
14	719	726.1	.070	11.386
15	571	525.8	3.879	15.265
16	376	381.2	.070	15.335
17	257	276.5	1.381	16.715
18	189	200.8	.697	17.412
19	156	146.0	.687	18.099
20	114	106.2	.571	18.670
21	308	287.1	1.519	20.189
SUMMARY	FOR block6	.rng		
p-val	ue for no.	of wins:	.828643	
p-val	ue for thr	ows/game:	.553823	

#### 

Results of DIEHARD battery of tests sent to file report6.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p > .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If j is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean  $m^3/(4n)$ . Experience shows n :: :: must be quite large, say  $n \ge 2^{18}$ , for comparing the results :: :: to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. : : :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: :: Each set of bits provides a p-value, and the nine p-values :: provide a sample for a KSTEST. :: BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for block7.rng For a sample of size 500: mean using bits 1 to 24 2.020 block7.rng duplicate number number

observed spacings expected 70. 67.668 0 135.335 1 131. 2 139. 135.335 3 88. 90.224 4 40. 45.112 5 21. 18.045 6 to INF 11. 8.282 Chisquare with 6 d.o.f. = 2.33 p-value= .112873 For a sample of size 500: mean block7.rng using bits 2 to 25 1.958 duplicate number number spacings observed expected 0 69. 67.668 135. 1 135.335 2 137. 135.335 3 89. 90.224 4 50. 45.112 5 18.045 17. 6 to INF 8.282 3. Chisquare with 6 d.o.f. =4.02 p-value= .326413 For a sample of size 500: mean block7.rng using bits 3 to 26 2.060 duplicate number number expected spacings observed 0 67. 67.668 1 129. 135.335 2 143. 135.335 3 81. 90.224 4 47. 45.112 5 18. 18.045 6 to INF 15. 8.282 Chisquare with 6 d.o.f. =7.21 p-value= .698057 For a sample of size 500: mean using bits 4 to 27 block7.rng 1.970 number duplicate number spacings observed expected 79. 67.668 0 1 127. 135.335 2 126. 135.335 3 96. 90.224 4 51. 45.112 5 18.045 13. 8.282 6 to INF 8. Chisquare with 6 d.o.f. = 5.61 p-value= .532150 For a sample of size 500: mean block7.rng using bits 5 to 28 2.050 duplicate number number spacings observed expected 0 58. 67.668 1 141. 135.335 2 139. 135.335 3 90.224 85.

4 46. 45.112 5 21. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =2.88 p-value= .175996 For a sample of size 500: mean block7.rng using bits 6 to 29 2.014 duplicate number number expected spacings observed 0 65. 67.668 1 134. 135.335 2 132. 135.335 3 87. 90.224 4 69. 45.112 5 7. 18.045 6 to INF б. 8.282 Chisquare with 6 d.o.f. = 20.35 p-value= .997605 For a sample of size 500: mean using bits 7 to 30 1.938 block7.rng duplicate number number spacings observed expected 0 72. 67.668 1 132. 135.335 2 142. 135.335 3 92. 90.224 4 45.112 41. 5 15. 18.045 6 to INF 6. 8.282 Chisquare with 6 d.o.f. = 2.24 p-value= .103639 For a sample of size 500: mean block7.rng using bits 8 to 31 1.948 duplicate number number spacings observed expected 0 57. 67.668 1 150. 135.335 2 142. 135.335 3 97. 90.224 4 30. 45.112 5 19. 18.045 6 to INF 5. 8.282 Chisquare with 6 d.o.f. = 10.52 p-value= .895650 ..... For a sample of size 500: mean block7.rng using bits 9 to 32 1.978 duplicate number number spacings observed expected 0 65. 67.668 1 142. 135.335 2 138. 135.335 3 83. 90.224 4 44. 45.112 5 24. 18.045 6 to INF 4. 8.282 5.27 p-value= Chisquare with 6 d.o.f. =.490415 

The 9 p-values	were		
.112873	.326413 .698057	.532150	.175996
.997605	.103639 .895650	.490415	
A KSTEST for the	e 9 p-values yields	.282488	

\$

:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: OPERM5 test for file block7.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom= 79.893; p-value= .079479 OPERM5 test for file block7.rng For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=114.672; p-value= .865868 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for block7.rng Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer rank observed expected  $(o-e)^2/e$  sum 28 222 211.4 .529654 .530 29 5258 5134.0 2.994434 3.524 30 22990 23103.0 .553156 4.077 31 11530 11551.5 .040107 4.117 chisquare= 4.117 for 3 d. of f.; p-value= .772155 :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. : : Binary rank test for block7.rng

Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer observed expected (o-e)^2/e sum rank 211.4 1.813725 1.814 29 231 .296104 30 5173 5134.0 2.110 .495997 31 22996 23103.0 2.606 32 11600 11551.5 .203426 2.809 chisquare= 2.809 for 3 d. of f.; p-value= .624882 \_\_\_\_\_ :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a  $\ ::$ :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: :: random matrices, and a chi-square test is performed on :: counts for ranks 6,5 and <=4. :: Binary Rank Test for block7.rng Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 943 944.3 .002 .002 r =5 21369 21743.9 6.464 6.466 77688 r =6 77311.8 1.831 8.296 p=1-exp(-SUM/2)=.98421Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 2 to 9 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 947 944.3 .008 .008 r =5 21792 21743.9 .106 .114 77261 77311.8 .147 r =б .033 p=1-exp(-SUM/2)=.07110Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 3 to 10 OBSERVED EXPECTED (O-E)^2/E SUM 1047 944.3 11.169 11.169 r<=4 r =5 21753 21743.9 .004 11.173 r =6 77200 77311.8 .162 11.335 p=1-exp(-SUM/2)=.99654Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .395 r<=4 925 944.3 .395 21690 21743.9 .134 r =5 .528 .069 .597 77385 77311.8 r =6 p=1-exp(-SUM/2)=.25823Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng

b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 879 944.3 4.516 4.516 r =5 21669 21743.9 .258 4.774 77452 77311.8 .254 r =6 5.028 p=1-exp(-SUM/2)=.91906Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 918 944.3 .733 .733 r =5 21808 21743.9 .189 .922 r =6 77274 77311.8 .018 .940 p=1-exp(-SUM/2)=.37500Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 883 944.3 3.979 3.979 r<=4 21599 21743.9 .966 4.945 r =5 77518 r =6 77311.8 .550 5.495 p=1-exp(-SUM/2)=.93591Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 928 944.3 .281 .281 r =5 21857 21743.9 .588 .870 r =6 77215 77311.8 .121 .991 p=1-exp(-SUM/2)=.39070Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 3.774 r<=4 1004 944.3 3.774 r =5 21616 21743.9 .752 4.526 77380 77311.8 .060 4.587 r =6 p=1-exp(-SUM/2)=.89907Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .002 .002 r<=4 943 944.3 21896 21743.9 1.064 1.066 r =5 77161 r =6 77311.8 .294 1.360 p=1-exp(-SUM/2)=.49336Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 907 944.3 1.473 1.473 r =5 21836 21743.9 .390 1.864 .039 r =6 77257 77311.8 1.902 p=1-exp(-SUM/2)=.61372Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG block7.rng b-rank test for bits 12 to 19

r<=4 r =5 r =6	941 21578 77481	944.3 21743.9 77311.8	.370	SUM .012 1.277 1.648
		(-SUM/2) = .5	56124	
	x8 binary ma		l blogla 7 mag	
rows formed fr b-rank test for			block/.rng	
D-IAIR CESC IO			(O-E)^2/E	SUM
r<=4		944.3		.199
r =5		21743.9	.031	.230
r =6		77311.8	.020	.251
	p=1-exp	(-SUM/2) = .1	1774	
	x8 binary ma	itrix,		
rows formed fr			B block7.rng	
b-rank test for			/ <b></b> /	
		EXPECTED	(O-E)^2/E	SUM
r<=4	958 21760	944.3	.199	.199
r =5			.012	.211
r =6		77311.8		.222
Pank of a 6	p=1-exp x8 binary ma	(-SUM/2) = .1	.0512	
rows formed fr			hlock7 rng	
b-rank test for			biock/.ing	
			(O-E)^2/E	SUM
r<=4		944.3	2.616	2.616
r =5		21743.9		3.443
r =6		77311.8	.437	3.880
	p=1-exp	(-SUM/2) = .8	35627	
	x8 binary ma			
rows formed fr			G block7.rng	
b-rank test for				
			(O-E)^2/E	SUM
r<=4		944.3		.160
r =5	21798	21743.9	.135	.295
r =6		77311.8	.023	.317
Deals of a C		(-SUM/2) = .1	46//	
rows formed fr	x8 binary ma		block7 mg	
b-rank test for			biock/.ing	
D TAIK CESC IO	OBSERVED		(O-E)^2/E	SUM
r<=4	984	944.3	1.669	1.669
r =5	21527	21743.9	2.164	3.833
r =6	77489	77311.8	.406	4.239
	p=1-exp	(-SUM/2) = .8	37989	
Rank of a 6	x8 binary ma	itrix,		
rows formed fr			B block7.rng	
b-rank test for	r bits 18 to	25		
	OBSERVED	EXPECTED		SUM
r<=4	983	944.3	1.586	1.586
r =5		21743.9	.803	2.388
r =6		77311.8		2.766
Deals of a		(-SUM/2) = .7	4915	
Rank of a 6 rows formed fr	x8 binary ma	ILTIX,	block7 mor	
b-rank test for			DIOCK/.rig	
D-LANK LEST IO				
	OBSERVED		(O-E)^2/E	SUM

	0.2.0	044 2	0.4.2	0.4.2
r<=4 r =5	938 21833	944.3 21743.9	.042 .365	.042 .407
r =6	77229	77311.8	.089	.407
1 -0		$\exp(-SUM/2) =$		. 190
Rank of a	6x8 binary	-		
			NG block7.rng	
b-rank test f				
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	949	944.3	.023	.023
r =5		21743.9	.020	.044
r =6	77286		.009	.052
Devile of a		$\exp(-SUM/2) =$	.02589	
	6x8 binary			
b-rank test f			NG block7.rng	
D-TAIL LEST I	OBSERVED		(O-E)^2/E	SUM
r<=4	999	944.3		3.168
r =5	21819		.259	3.428
r =6	77182	77311.8	.218	3.646
- •	-	$\exp(-SUM/2) =$		0.010
Rank of a	6x8 binary			
			NG block7.rng	
b-rank test f	for bits 22	to 29		
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	950	944.3	.034	.034
r =5	21826	21743.9	.310	.344
r =6	77224	77311.8	.100	.444
	-	$\exp(-SUM/2) =$	.19912	
	6x8 binary			
			NG block7.rng	
b-rank test f				OT TM
r<=4	OBSERVED 963	EXPECTED 944.3	(O-E)^2/E .370	SUM .370
r = 4 r = 5		21743.9		.370
r =6	77324		.002	.416
1 =0	-	$\exp(-SUM/2) =$		.110
Rank of a	6x8 binary		. 10,00	
			NG block7.rng	
b-rank test f			5	
	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	988	944.3	2.022	2.022
r =5	21864		.663	2.686
r =6	77148		.347	3.033
		$\exp(-SUM/2) =$	.78048	
	6x8 binary			
			NG block7.rng	
b-rank test f				
	OBSERVED	-	(O-E)^2/E	
r<=4	946 21617	944.3	.003	.003
r =5 r =6	77437	21743.9 77311.8	.741 .203	.744 .946
т -0		$\exp(-SUM/2) =$		.940
TEST SUMMARY, 2				es
These should be 2				
				919057
				493357
	561239	.117744		856272

.146771	.879891	.749153	.219577	.025893			
.838439	.199125	.187835	.780481	.376994			
brank test	summary for	block7.rng					
The KS	test for tho	se 25 supposed	UNI's yield	.S			
KS p-value= .558088							

#### \$

:: THE BITSTREAM TEST :: :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... . Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of 2^21+19 bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated :: :: twenty times. :: 

THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words

This test uses N=2^21 and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428.

tst n	10	1:	141621	missing	words,	67	sigmas	from	mean,	p-value=	.25026
tst n	10	2:	141950	missing	words,	.10	sigmas	from	mean,	p-value=	.53785
tst n	10	3:	141541	missing	words,	86	sigmas	from	mean,	p-value=	.19473
tst n	10	4:	142871	missing	words,	2.25	sigmas	from	mean,	p-value=	.98768
tst n	10	5:	141595	missing	words,	73	sigmas	from	mean,	p-value=	.23135
tst n	10	6:	141722	missing	words,	44	sigmas	from	mean,	p-value=	.33081
tst n	10	7:	141861	missing	words,	11	sigmas	from	mean,	p-value=	.45505
tst n	10	8:	142415	missing	words,	1.18	sigmas	from	mean,	p-value=	.88129
tst n	10	9:	141673	missing	words,	55	sigmas	from	mean,	p-value=	.29042
tst n	10	10:	141273	missing	words,	-1.49	sigmas	from	mean,	p-value=	.06854
tst n	10	11:	141683	missing	words,	53	sigmas	from	mean,	p-value=	.29847
tst n	10	12:	141061	missing	words,	-1.98	sigmas	from	mean,	p-value=	.02374
tst n	10	13:	142007	missing	words,	.23	sigmas	from	mean,	p-value=	.59026
tst n	10	14:	141330	missing	words,	-1.35	sigmas	from	mean,	p-value=	.08794
tst n	10	15:	142562	missing	words,	1.52	sigmas	from	mean,	p-value=	.93636
tst n	10	16:	142229	missing	words,	.75	sigmas	from	mean,	p-value=	.77244
tst n	10	17:	142178	missing	words,	.63	sigmas	from	mean,	p-value=	.73491
tst n	10	18:	142172	missing	words,	.61	sigmas	from	mean,	p-value=	.73030
tst n	10	19:	141751	missing	words,	37	sigmas	from	mean,	p-value=	.35572
tst n	10	20:	142016	missing	words,	.25	sigmas	from	mean,	p-value=	.59841

:: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: be a standard normal variable. The OPSO test takes 32 bits at :: :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: file, elements of which are assumed 32-bit random integers. :: :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2<sup>21+3</sup> "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: :: extensive simulation. :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: determined by two designated bits in the sequence of random :: integers being tested. It considers 10-letter words, so that :: :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: :: mean number of missing words from a string of 2^21 (over-:: :: lapping) 10-letter words (2^21+9 "keystrokes") is 141909. :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: three places), not determined by simulation. :: OPSO test for generator block7.rng Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р OPSO for block7.rng using bits 23 to 32 142031 .420 .6626 OPSO for block7.rng using bits 22 to 31 141892 -.060 .4762 OPSO for block7.rng using bits 21 to 30 142042 .457 .6763 OPSO for block7.rng using bits 20 to 29 141920 .037 .5147 OPSO for block7.rng using bits 19 to 28 142114 .706 .7598 OPSO for block7.rng using bits 18 to 27 142297 1.337 .9094 OPSO for block7.rng using bits 17 to 26 142336 1.471 .9294 .092 OPSO for block7.rng using bits 16 to 25 141936 .5366 OPSO for block7.rng using bits 15 to 24 142349 1.516 .9353 OPSO for block7.rng using bits 14 to 23 141599 -1.070 .1423 OPSO for block7.rng using bits 13 to 22 141507 -1.387 .0827 OPSO for block7.rng using bits 12 to 21 141716 -.667 .2525 OPSO for block7.rng using bits 11 to 20 141735 -.601 .2739 OPSO for block7.rng using bits 10 to 19 141866 -.149 .4406 OPSO for block7.rng using bits 9 to 18 141789 -.415 .3391 OPSO for block7.rng using bits 8 to 17 142165 .882 .8110 OPSO for block7.rng using bits 7 to 16 142153 .840 .7996 OPSO for block7.rng using bits 6 to 15 141699 -.725 .2341 OPSO for block7.rng using bits 5 to 14 -.784 141682 .2166 OPSO for block7.rng using bits 4 to 13 142174 .913 .8193 using bits .685 OPSO for block7.rng 3 to 12 142108 .7534 using bits OPSO for block7.rng 141767 -.491 2 to 11 .3118 .557 OPSO for block7.rng using bits 1 to 10 142071 .7114

OQSO test for generator block7.rng

Output: No. missing words	(mw), equiv normal variate	e (z), p-value (p)
		mw z p
OQSO for block7.rng	using bits 28 to 32	141417 -1.669 .0476
OQSO for block7.rng	using bits 27 to 31	141885082 .4671
OQSO for block7.rng	using bits 26 to 30	141679781 .2175
OQSO for block7.rng	using bits 25 to 29	142498 1.995 .9770
OQSO for block7.rng	using bits 24 to 28	142383 1.606 .9458
OQSO for block7.rng	using bits 23 to 27	141743564 .2864
OQSO for block7.rng	using bits 22 to 26	141650879 .1897 142042 .450 .6735
OQSO for block7.rng	using bits 21 to 25	142042 .450 .6735 142112 .687 .7540
OQSO for block7.rng OQSO for block7.rng	using bits 20 to 24 using bits 19 to 23	142112 .007 .7540
OQSO for block7.rng	using bits 18 to 22	142322 1.399 .9191
OQSO for block7.rng	using bits 17 to 21	142163 .860 .8051
OQSO for block7.rng	using bits 16 to 20	141799374 .3542
OQSO for block7.rng	using bits 15 to 19	141851198 .4216
OQSO for block7.rng	using bits 14 to 18	141496 -1.401 .0806
OQSO for block7.rng	using bits 13 to 17	141855184 .4269
OQSO for block7.rng	using bits 12 to 16	141909001 .4996
OQSO for block7.rng	using bits 11 to 15	141766486 .3135
OQSO for block7.rng	using bits 10 to 14	141374 -1.815 .0348
OQSO for block7.rng	using bits 9 to 13	141707686 .2464
OQSO for block7.rng	using bits 8 to 12	141973 .216 .5854
OQSO for block7.rng	using bits 7 to 11	142142 .789 .7849
OQSO for block7.rng	using bits 6 to 10	141584 -1.103 .1351
OQSO for block7.rng	using bits 5 to 9	141951 .141 .5562
OQSO for block7.rng	using bits 4 to 8	141504 -1.374 .0847
OQSO for block7.rng	using bits 3 to 7	141547 -1.228 .1097
		141620 040 1026
OQSO for block7.rng	using bits 2 to 6	141632940 .1736
OQSO for block7.rng	using bits 1 to 5	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
OQSO for block7.rng DNA test for generator blo	using bits 1 to 5 ock7.rng	141813327 .3720
OQSO for block7.rng DNA test for generator blo	using bits 1 to 5	141813327 .3720 e (z), p-value (p)
OQSO for block7.rng DNA test for generator blo Output: No. missing words	using bits 1 to 5 ock7.rng (mw), equiv normal variate	141813327 .3720 e (z), p-value (p) mw z p
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 23 to 24 using bits 22 to 23 using bits 21 to 22	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 23 to 24 using bits 22 to 23 using bits 21 to 22 using bits 20 to 21	141813327 .3720 (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 ock7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 26 to 27 using bits 26 to 27 using bits 24 to 25 using bits 23 to 24 using bits 22 to 23 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210 142100 .562 .7131
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 23 to 24 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 18 to 19	141813327 .3720 (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210 142100 .562 .7131 141539 -1.092 .1373
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 23 to 24 using bits 22 to 23 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 18 to 19 using bits 17 to 18	141813327 .3720 (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210 142100 .562 .7131 141539 -1.092 .1373 141819266 .3949
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 22 to 23 using bits 21 to 22 using bits 19 to 20 using bits 18 to 19 using bits 16 to 17	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210 142100 .562 .7131 141539 -1.092 .1373 141819266 .3949 141949 .117 .5466
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 17 to 18 using bits 15 to 16	141813327 .3720 e (z), p-value (p) mw z p 141220 -2.033 .0210 141515 -1.163 .1224 141450 -1.355 .0877 141584960 .1686 141795337 .3680 142217 .908 .8180 141548 -1.066 .1432 141503 -1.199 .1153 142025 .341 .6335 141775396 .3460 141777390 .3481 142388 1.412 .9210 142100 .562 .7131 141539 -1.092 .1373 141819266 .3949 141949 .117 .5466 141814281 .3893
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 17 to 18 using bits 15 to 16 using bits 14 to 15	141813327 .3720 (z), p-value (p) mw $z$ $p141220 -2.033 .0210141515 -1.163 .1224141450 -1.355 .0877141584960 .1686141795337 .3680142217 .908 .8180141548 -1.066 .1432141503 -1.199 .1153142025 .341 .6335141775396 .3460141777390 .3481142388 1.412 .9210142100 .562 .7131141539 -1.092 .1373141819266 .3949141949 .117 .5466141814281 .3893141894045 .4820$
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 26 to 27 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 18 to 19 using bits 16 to 17 using bits 15 to 16 using bits 14 to 15 using bits 13 to 14	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 26 to 27 using bits 26 to 27 using bits 24 to 25 using bits 23 to 24 using bits 21 to 22 using bits 19 to 20 using bits 18 to 19 using bits 15 to 16 using bits 14 to 15 using bits 12 to 13	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 16 to 17 using bits 15 to 16 using bits 13 to 14 using bits 12 to 13 using bits 11 to 12	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 23 to 24 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 16 to 17 using bits 15 to 16 using bits 13 to 14 using bits 12 to 13 using bits 10 to 11	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
OQSO for block7.rng DNA test for generator blo Output: No. missing words DNA for block7.rng DNA for block7.rng	using bits 1 to 5 pck7.rng (mw), equiv normal variate using bits 31 to 32 using bits 30 to 31 using bits 29 to 30 using bits 28 to 29 using bits 27 to 28 using bits 26 to 27 using bits 25 to 26 using bits 24 to 25 using bits 21 to 22 using bits 20 to 21 using bits 19 to 20 using bits 16 to 17 using bits 15 to 16 using bits 13 to 14 using bits 12 to 13 using bits 11 to 12	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

DNA for b	lock7.rng	using	bits	7	to	8	141514	-1.166	.1218
DNA for b	lock7.rng	using	bits	б	to	7	141890	057	.4773
DNA for b	lock7.rng	using	bits	5	to	6	141752	464	.3213
DNA for b	lock7.rng	using	bits	4	to	5	142441	1.568	.9416
DNA for b	lock7.rng	using	bits	3	to	4	141630	824	.2050
DNA for b	lock7.rng	using	bits	2	to	3	142319	1.208	.8866
DNA for b	lock7.rng	using	bits	1	to	2	141620	853	.1967

:: This is the COUNT-THE-1's TEST on a stream of bytes. :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5<sup>5</sup> :: :: possible 5-letter words, and from a string of 256,000 (over-:: :: lapping) 5-letter words, counts are made on the frequencies :: :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts. :: Test results for block7.rng

Chi-square with 5^5-5^4=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for block7.rng 2487.17 -.181 .427992

byte	stream	for	block7.rng	2	2655.38	2.197	.986004

This is the COUNT-THE-1's TEST for specific bytes. :: :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left- :: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: 

Chi-square with	5^5-				
				uiv normal p	value
Results for CO	UNT-TI	HE-1	's in spec	ified bytes:	
bits	1 to	8	2671.31	2.423	.992296
bits	2 to	9	2530.80	.436	.668404
bits	3 to	10	2459.45	573	.283184
bits	4 to	11	2560.92	.861	.805514
bits	5 to	12	2559.04	.835	.798121
bits	6 to	13	2602.63	1.451	.926676
bits	7 to	14	2359.06	-1.993	.023123
bits	8 to	15	2525.92	.367	.643040
bits	9 to	16	2403.80	-1.360	.086846
bits	10 to	17	2453.50	658	.255385
bits	11 to	18	2441.18	832	.202762
bits	12 to	19	2569.72	.986	.837940
bits	13 to	20	2555.82	.789	.785081
bits	14 to	21	2416.67	-1.178	.119306
bits	15 to	22	2665.60	2.342	.990409
bits	16 to	23	2637.61	1.946	.974176
bits	17 to	24	2583.10	1.175	.880045
bits	18 to	25	2505.24	.074	.529563
bits	19 to	26	2498.78	017	.493110
bits	20 to	27	2429.44	998	.159179
bits	21 to	28	2578.95	1.117	.867904
bits	22 to	29	2422.27	-1.099	.135822
bits	23 to	30	2560.72	.859	.804757
bits	24 to	31	2447.31	745	.228083
bits	25 to	32	2623.04	1.740	.959080

:: THIS IS A PARKING LOT TEST :: :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect :: :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: :: to normally distributed. Thus (k-3523)/21.9 should be a st-:: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file block7.rng Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 Successes: 3576 z-score: 2.420 p-value: .992242

Successes:	3487	z-score:	-1.644	p-value:	.050105
Successes:	3586	z-score:	2.877	p-value:	.997991
Successes:	3502	z-score:	959	p-value:	.168804
Successes:	3501	z-score:	-1.005	p-value:	.157553
Successes:	3533	z-score:	.457	p-value:	.676028
Successes:	3502	z-score:	959	p-value:	.168804
Successes:	3552	z-score:	1.324	p-value:	.907282
Successes:	3555	z-score:	1.461	p-value:	.928018
Successes:	3504	z-score:	868	p-value:	.192812
square size	avg.	no. parke	ed sai	mple sigma	a
100		2520 000	22	621	

100.3529.80033.621KSTEST for the above 10: p=.942137

:: THE MINIMUM DISTANCE TEST ::
:: square of side 10000. Find d, the minimum distance between ::
:: the $(n^2-n)/2$ pairs of points. If the points are truly inde- ::
:: pendent uniform, then d^2, the square of the minimum distance ::
:: should be (very close to) exponentially distributed with mean :: :: .995 . Thus $1 - \exp(-d^2/.995)$ should be uniform on [0.1) and ::
()))) in ab i onp( a i, ()))), bhoaid bo aniiltein on [o,i, and
:: a KSTEST on the resulting 100 values serves as a test of uni- ::
:: formity for random points in the square. Test numbers=0 mod 5 ::
<pre>:: are printed but the KSTEST is based on the full set of 100 :: :: random choices of 8000 points in the 10000x10000 square. ::</pre>
:: random choices of 8000 points in the 10000x10000 square. ::
This is the MINIMUM DISTANCE test
for random integers in the file block7.rng
Sample no. d^2 avg equiv uni 5 .6298 1.2579 .468992
10   .4375   1.0111   .355762
15 .2750 .9661 .241502
20 1.8928 .8954 .850770
25 .3661 .7976 .307844
30 .5591 .7787 .429891
35 1.1869 .7616 .696662
40 2.4796 .8077 .917257
45 .5980 .8196 .451718
50 .7859 .8383 .546078
55 .2071 .8181 .187945
60 .9889 .8003 .629864
65 .4289 .7858 .350155
70 .7257 .7641 .517774
75 2.2272 .7788 .893371
80 .1968 .7566 .179480
85 .0156 .7603 .015588
90 .3938 .7972 .326867
95 1.6271 .7931 .805103
100 .1752 .8205 .161470
MINIMUM DISTANCE TEST for block7.rng
Result of KS test on 20 transformed mindist^2's:
p-value= .739882

:: THE 3DSPHERES TEST :: Choose 4000 random points in a cube of edge 1000. At each :: :: point, center a sphere large enough to reach the next closest :: :: point. Then the volume of the smallest such sphere is (very :: :: close to) exponentially distributed with mean 120pi/3. Thus :: :: :: the radius cubed is exponential with mean 30. (The mean is :: obtained by extensive simulation). The 3DSPHERES test gener- :: :: ates 4000 such spheres 20 times. Each min radius cubed leads :: :: to a uniform variable by means of  $1-\exp(-r^3/30.)$ , then a :: :: KSTEST is done on the 20 p-values. :: The 3DSPHERES test for file block7.rng r^3= 37.433 sample no: 1 p-value= .71286 r^3= sample no: 2 .306 p-value= .01016 r^3= 43.498 p-value= .76541 sample no: 3 sample no: 4 r^3= 9.718 p-value= .27669 sample no: 5 r^3= 11.180 p-value= .31110 sample no: 6 r^3= 48.293 p-value= .80007 sample no: 7 r^3= 17.462 p-value= .44126 r^3= 55.525 sample no: 8 p-value= .84290 r^3= 27.461 sample no: 9 p-value= .59964 r^3= 62.657 sample no: 10 p-value= .87614 sample no: 11 r^3= 4.779 p-value= .14726 p-value= .77109 sample no: 12 r^3= 44.232 sample no: 13 r^3= 13.391 p-value= .36006 r^3= 16.130 sample no: 14 p-value= .41590 sample no: 15 r^3= 35.358 p-value= .69229 sample no: 16 r^3= 7.551 p-value= .22253 r^3= 108.515 sample no: 17 p-value= .97314 r^3= 6.018 sample no: 18 p-value= .18175 r^3= 44.436 sample no: 19 p-value= .77263 r^3= 5.681 sample no: 20 p-value= .17253 A KS test is applied to those 20 p-values. -----3DSPHERES test for file block7.rng p-value= .098354 :: This is the SQEEZE test : : :: Random integers are floated to get uniforms on [0,1). Start- :: :: ing with k=2^31=2147483647, the test finds j, the number of :: :: iterations necessary to reduce k to 1, using the reduction :: :: k=ceiling(k\*U), with U provided by floating integers from :: :: the file being tested. Such j's are found 100,000 times, :: :: then counts for the number of times j was  $<=6,7,\ldots,47,>=48$  :: :: are used to provide a chi-square test for cell frequencies. :: RESULTS OF SQUEEZE TEST FOR block7.rng Table of standardized frequency counts ( (obs-exp)/sqrt(exp) )^2 for j taking values <=6,7,8,...,47,>=48: 1.8 -.4 -.8 1.2 .0 -1.5 .9 -.7 1.7 .9 .2 .1 .5 -.4 -.2 -.4 -.6 -1.1

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.1

-.5

.2

-.2

.9

-1.0

1.7 1.1 .6 -.б -.8 -.4 -1.7 -1.7 -.9 .3 -.3 -1.6 -1.4-1.7 -2.0 -.7 -.6 1.0 -1.1 Chi-square with 42 degrees of freedom: 43.734 z-score= .189 p-value= .602332 :: The OVERLAPPING SUMS test :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: Test no. 1 p-value .676187 Test no. 2 p-value .186294 Test no. 3 p-value .022401 Test no. 4 p-value .617651 Test no. 5 p-value .544124 Test no. б p-value .302474 Test no. 7 p-value .557463 Test no. 8 p-value .802135 .411520 Test no. 9 p-value Test no. 10 p-value .563264 Results of the OSUM test for block7.rng KSTEST on the above 10 p-values: .378851 :: This is the RUNS test. It counts runs up, and runs down, :: :: in a sequence of uniform [0,1) variables, obtained by float- :: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file block7.rng Up and down runs in a sample of 10000 Run test for block7.rng :

runs up; ks test for 10 p's: .096866 runs down; ks test for 10 p's: .802117 Run test for block7.rng : runs up; ks test for 10 p's: .563537

runs down; ks test for 10 p's: .400210

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:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: :: normal with mean 200000p and variance 200000p(1-p), with :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for block7.rng No. of wins: Observed Expected 98425 98585.86 98425= No. of wins, z-score= -.719 pvalue= .23593 Analysis of Throws-per-Game: Chisq= 17.64 for 20 degrees of freedom, p= .38884 Throws Observed Expected Chisq Sum 66636 66666.7 .014 1 .014 2 37642 37654.3 .004 .018 27016 26954.7 3 .139 .157 .352 4 19231 19313.5 .509 5 14029 13851.4 2.277 2.786 6 9950 9943.5 .004 2.790 .988 7 7061 7145.0 3.778 2.231 8 5032 5139.1 6.009 3699.9 9 3743 6.512 .503 10 2708 2666.3 .652 7.164 11 1873 1923.3 1.317 8.481 12 8.501 1394 1388.7 .020 13 1009 1003.7 .028 8.529 14 740 726.1 8.794 .265 15 490 525.8 2.442 11.236 1.492 16 405 381.2 12.728 17 280 276.5 .043 12.772 18 200.8 13.917 216 1.146 19 141 146.0 .170 14.088 20 125 106.2 3.322 17.410 21 279 287.1 .229 17.639 SUMMARY FOR block7.rng p-value for no. of wins: .235930 p-value for throws/game: .388841

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Results of DIEHARD battery of tests sent to file report7.txt

NOTE: Most of the tests in DIEHARD return a p-value, which should be uniform on [0,1) if the input file contains truly

independent random bits. Those p-values are obtained by p=F(X), where F is the assumed distribution of the sample random variable X---often normal. But that assumed F is just an asymptotic approximation, for which the fit will be worst in the tails. Thus you should not be surprised with occasional p-values near 0 or 1, such as .0012 or .9983. When a bit stream really FAILS BIG, you will get p's of 0 or 1 to six or more places. By all means, do not, as a Statistician might, think that a p < .025 or p > .975 means that the RNG has "failed the test at the .05 level". Such p's happen among the hundreds that DIEHARD produces, even with good RNG's. So keep in mind that " p happens". :: This is the BIRTHDAY SPACINGS TEST :: :: Choose m birthdays in a year of n days. List the spacings :: :: between the birthdays. If  ${\tt j}$  is the number of values that :: :: occur more than once in that list, then j is asymptotically :: :: Poisson distributed with mean m<sup>3</sup>/(4n). Experience shows n :: :: must be quite large, say n>=2^18, for comparing the results ::  $\colon\colon$  to the Poisson distribution with that mean. This test uses :: ::  $n=2^{24}$  and  $m=2^{9}$ , so that the underlying distribution for j :: :: is taken to be Poisson with  $lambda=2^{27}/(2^{26})=2$ . A sample :: :: of 500 j's is taken, and a chi-square goodness of fit test :: :: provides a p value. The first test uses bits 1-24 (counting :: :: from the left) from integers in the specified file. :: :: Then the file is closed and reopened. Next, bits 2-25 are :: :: used to provide birthdays, then 3-26 and so on to bits 9-32. :: :: Each set of bits provides a p-value, and the nine p-values :: :: provide a sample for a KSTEST. : : BIRTHDAY SPACINGS TEST, M= 512 N=2\*\*24 LAMBDA= 2.0000 Results for BLOCKX.RNG For a sample of size 500: mean BLOCKX.RNG using bits 1 to 24 1.888 duplicate number number observed expected spacings 0 74. 67.668 1 162. 135.335 2 122. 135.335 3 75. 90.224 4 36. 45.112 5 21. 18.045 6 to INF 10. 8.282 Chisquare with 6 d.o.f. =12.41 p-value= .946574 For a sample of size 500: mean BLOCKX.RNG using bits 2 to 25 1.990 number duplicate number observed expected spacings 0 66. 67.668 1 138. 135.335 2 137. 135.335 3 95. 90.224 4 35. 45.112 5 18 18.045 11. 6 to INF 8.282 Chisquare with 6 d.o.f. = 3.53 p-value= .259454

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•	•	٠	•	•	•	٠	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	٠	•	•	•	٠	٠	•	٠	•	•	•	

		ample of dide FOO:	
דם	OCKX.RNG	ample of size 500: using bits 3 to 26	1 000
duplicate	number	number	1.900
-	observed		
o o spacings	74.	67.668	
1	131.	135.335	
2	141.	135.335	
3	81.	90.224	
4		45.112	
5	22.	18.045	
6 to INF	22. 9.	8.282	
		= 3.06 p-value=	100126
		- 5.00 p varue-	.190190
		ample of size 500:	moon
BI.	OCKX.RNG	using bits 4 to 27	
	number		2.100
spacings	observed		
o o spacings	61.	67.668	
1	129.	135.335	
1 2	136.	135.335	
3	92.	90.224	
4	92. 46.		
5	40. 24.	45.112	
5 6 to INF	12.	18.045 8.282	
	⊥4. th Calaf	<pre>o.2o2 = 4.64 p-value=</pre>	400700
		= 4.64 p-value=	.409792
		ample of size 500:	moon
דת	OCKX.RNG	using bits 5 to 28	
	OCKA.KNG	using bits 5 to 20	1.944
duplicate spacings	number observed	number expected	
o o spacings	73.	67.668	
1	135.	135.335	
2	142.		
3	77.	90.224	
4	50.	45.112	
5	17.	18.045	
6 to INF	۲, 6.	8.282	
		= 3.91 p-value=	310630
		·····	.910090
		ample of size 500:	mean
BL		using bits 6 to 29	
duplicate	number	number	11710
spacings	observed	expected	
0	86.	67.668	
1	125.	135.335	
2	133.	135.335	
3	76.	90.224	
4	53.	45.112	
5	22.	18.045	
6 to INF	5.	8.282	
6 to INF Chisquare wi	5. th 6 d.o.f. :	8.282 = 11.59 p-value=	.928112
Chisquare wi	th 6 d.o.f. :		.928112
Chisquare wi	th 6 d.o.f. :	= 11.59 p-value=	
Chisquare wi	th 6 d.o.f. : ::::::::: For a sa	= 11.59 p-value= ::::::::::::::::::::::::::::::::::::	.928112 mean 1.964
Chisquare wi :::::::::: BL	th 6 d.o.f. :	= 11.59 p-value=	mean
Chisquare wi	th 6 d.o.f. = :::::::: For a sa OCKX.RNG	= 11.59 p-value= ::::::::::::::::::::::::::::::::::::	mean

0 74. 67.668 135.335 1 131. 2 135.335 134. 3 92. 90.224 4 45.112 46. 5 16. 18.045 7. 8.282 6 to INF 1.23 p-value= .024479 Chisquare with 6 d.o.f. =For a sample of size 500: mean using bits 8 to 31 BLOCKX.RNG 2.094 duplicate number number observed expected spacings 0 57. 67.668 1 122. 135.335 2 142. 135.335 3 90.224 113. 4 40. 45.112 5 16. 18.045 8.282 10. 6 to INF Chisquare with 6 d.o.f. = 10.24 p-value= .885142 For a sample of size 500: mean BLOCKX.RNG using bits 9 to 32 1.956 duplicate number number observed expected spacings 0 62. 67.668 1 145. 135.335 2 142. 135.335 3 87. 90.224 4 42. 45.112 5 14. 18.045 8. 6 to INF 8.282 Chisquare with 6 d.o.f. = 2.74 p-value= .159196 The 9 p-values were .259454 .409792 .310630 .946574 .198136 .928112 .024479 .885142 .159196 A KSTEST for the 9 p-values yields .551415

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:: THE OVERLAPPING 5-PERMUTATION TEST :: :: This is the OPERM5 test. It looks at a sequence of one mill- :: :: ion 32-bit random integers. Each set of five consecutive :: :: integers can be in one of 120 states, for the 5! possible or- :: :: derings of five numbers. Thus the 5th, 6th, 7th,...numbers :: :: each provide a state. As many thousands of state transitions :: :: are observed, cumulative counts are made of the number of :: :: occurences of each state. Then the quadratic form in the :: :: weak inverse of the 120x120 covariance matrix yields a test :: :: equivalent to the likelihood ratio test that the 120 cell :: :: counts came from the specified (asymptotically) normal dis-:: :: tribution with the specified 120x120 covariance matrix (with :: :: rank 99). This version uses 1,000,000 integers, twice. :: 

OPERM5 test for file BLOCKX.RNG For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom=100.986; p-value= .574389 OPERM5 test for file BLOCKX.RNG For a sample of 1,000,000 consecutive 5-tuples, chisquare for 99 degrees of freedom= 87.084; p-value= .201601 :: This is the BINARY RANK TEST for 31x31 matrices. The leftmost :: :: 31 bits of 31 random integers from the test sequence are used :: :: to form a 31x31 binary matrix over the field  $\{0,1\}$ . The rank :: :: is determined. That rank can be from 0 to 31, but ranks< 28 :: :: are rare, and their counts are pooled with those for rank 28. :: :: Ranks are found for 40,000 such random matrices and a chisqua-:: :: re test is performed on counts for ranks 31,30,29 and <=28. :: Binary rank test for BLOCKX.RNG Rank test for 31x31 binary matrices: rows from leftmost 31 bits of each 32-bit integer rank observed expected (o-e)^2/e sum 28 216 211.4 .099304 .099 .411 5174 29 5134.0 .311487 30 23171 23103.0 .199871 .611 11439 11551.5 1.096110 1.707 31 chisquare= 1.707 for 3 d. of f.; p-value= .460274 \_\_\_\_\_ :: This is the BINARY RANK TEST for 32x32 matrices. A random 32x :: :: 32 binary matrix is formed, each row a 32-bit random integer. :: :: The rank is determined. That rank can be from 0 to 32, ranks :: :: less than 29 are rare, and their counts are pooled with those :: :: for rank 29. Ranks are found for 40,000 such random matrices :: :: and a chisquare test is performed on counts for ranks 32,31, :: :: 30 and <=29. :: Binary rank test for BLOCKX.RNG Rank test for 32x32 binary matrices: rows from leftmost 32 bits of each 32-bit integer rank observed expected (o-e)^2/e sum .634489 .634 29 223 211.4 .359976 .994 30 5177 5134.0 23103.0 .166133 31 23165 1.161 11551.5 1.175424 32 11435 2.336 chisquare= 2.336 for 3 d. of f.; p-value= .557551 \_\_\_\_\_ :: This is the BINARY RANK TEST for 6x8 matrices. From each of :: :: six random 32-bit integers from the generator under test, a :: :: specified byte is chosen, and the resulting six bytes form a  $\ ::$ :: 6x8 binary matrix whose rank is determined. That rank can be :: :: from 0 to 6, but ranks 0,1,2,3 are rare; their counts are :: :: pooled with those for rank 4. Ranks are found for 100,000 :: :: :: random matrices, and a chi-square test is performed on :: counts for ranks 6,5 and <=4. :: 

Binary Rank Test for BLOCKX.RNG Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 1 to 8 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .909 r<=4 915 944.3 .909 .365 21833 21743.9 1.274 r =5 77252 77311.8 .046 1.321 r =6 p=1-exp(-SUM/2)=.48330Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 2 to 9 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM 924 944.3 .436 r<=4 .436 r =5 21728 21743.9 .012 .448 r =б 77348 77311.8 .017 .465 p=1-exp(-SUM/2)=.20746Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 3 to 10 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 891 944.3 3.009 3.009 r =5 21443 21743.9 4.164 7.173 r =6 77666 77311.8 1.623 8.795 p=1-exp(-SUM/2)=.98769Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 4 to 11 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .281 .281 r<=4 928 944.3 21523 21743.9 2.244 2.526 r =5 77549 r =б 77311.8 .728 3.253 p=1-exp(-SUM/2)=.80341Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 5 to 12 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 872 944.3 5.536 r<=4 5.536 21743.9 r =5 21476 3.301 8.837 r =б 77652 77311.8 1.497 10.333 p=1-exp(-SUM/2)=.99430Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 6 to 13 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .171 r<=4 957 944.3 .171 21744 r =5 21743.9 .000 .171 .173 77299 77311.8 .002 r =6 p=1-exp(-SUM/2)=.08281Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 7 to 14 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 1008 944.3 4.297 4.297 r < = 421606 21743.9 .875 r =5 5.171 77386 77311.8 .071 5.243 r =б p=1-exp(-SUM/2)=.92729

Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 8 to 15 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 939 944.3 .030 .030 r =5 21850 21743.9 .518 .547 .131 .679 77211 77311.8 r =6 p=1-exp(-SUM/2)=.28784Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 9 to 16 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .020 r<=4 940 944.3 .020 r =5 22067 21743.9 4.801 4.821 r =6 76993 77311.8 1.315 6.135 p=1-exp(-SUM/2)=.95347Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 10 to 17 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 932 944.3 .160 .160 r =5 21568 21743.9 1.423 1.583 r =6 77500 77311.8 .458 2.041 p=1-exp(-SUM/2)=.63964Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 11 to 18 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 910 944.3 1.246 1.246 .033 r =5 21717 21743.9 1.279 77373 77311.8 .048 r =6 1.328 p=1-exp(-SUM/2)=.48513Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 12 to 19 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 .395 .395 925 944.3 r =5 21645 21743.9 .450 .844 r =6 77430 77311.8 .181 1.025 p=1-exp(-SUM/2)=.40102Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 13 to 20 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 976 944.3 1.064 1.064 .110 r =5 21695 21743.9 1.174 .004 r =б 77329 77311.8 1.178 p=1-exp(-SUM/2)=.44508Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 14 to 21 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 924 944.3 .436 .436 .145 .581 r =5 21800 21743.9 77311.8 .017 .598 r =б 77276 p=1-exp(-SUM/2)=.25836Rank of a 6x8 binary matrix,

rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 15 to 22 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 923 944.3 .481 .481 21728 21743.9 .492 r =5 .012 .018 77349 77311.8 .510 r =6 p=1-exp(-SUM/2)=.22510Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 16 to 23 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 944.3 .281 928 . 281 r =5 21924 21743.9 1.492 1.773 77148 77311.8 .347 2.120 r =б p=1-exp(-SUM/2)=.65358Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 17 to 24 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 921 944.3 .575 .575 r =5 21812 21743.9 .213 .788 77267 77311.8 .814 r =6 .026 p=1-exp(-SUM/2)=.33443Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 18 to 25 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 938 944.3 .042 .042 r =5 21776 21743.9 .047 .089 .009 .098 r =6 77286 77311.8 p=1-exp(-SUM/2)=.04784Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 19 to 26 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 910 944.3 1.246 1.246 r =5 .005 21733 21743.9 1.251 77357 77311.8 1.278 r =б .026 p=1-exp(-SUM/2)=.47214Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 20 to 27 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM 884 944.3 3.851 3.851 r<=4 r =5 21776 21743.9 .047 3.898 r =6 77340 77311.8 .010 3.908 p=1-exp(-SUM/2)=.85832Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 21 to 28 EXPECTED OBSERVED  $(O-E)^{2}/E$ SUM r<=4 928 944.3 .281 .281 21661 21743.9 .597 r =5 .316 .127 .725 77411 77311.8 r =6 p=1-exp(-SUM/2)=.30398Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG

b-rank test for bits 22 to 29 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 930 944.3 .217 .217 r =5 21498 21743.9 2.781 2.997 77572 77311.8 .876 r =6 3.873 p=1-exp(-SUM/2)=.85580Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 23 to 30 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .034 r<=4 950 944.3 .034 r =5 21693 21743.9 .119 .154 r =6 77357 77311.8 .026 .180 p=1-exp(-SUM/2)=.08605Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 24 to 31 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM .034 r<=4 950 944.3 .034 21769 21743.9 .029 .063 r =5 r =6 77281 77311.8 .012 .076 p=1-exp(-SUM/2)=.03711Rank of a 6x8 binary matrix, rows formed from eight bits of the RNG BLOCKX.RNG b-rank test for bits 25 to 32 OBSERVED EXPECTED  $(O-E)^{2}/E$ SUM r<=4 961 944.3 .295 .295 r =5 21728 21743.9 .012 .307 r =б 77311 77311.8 .000 .307 p=1-exp(-SUM/2)=.14227TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices These should be 25 uniform [0,1] random variables: .483296 .207460 .987694 .803413 .994297 .927293 .082814 .287842 .953469 .639645 .485133 .401019 .445083 .258356 .225095 .334429 .047842 .472143 .653577 .858322 .303976 .855804 .086053 .037113 .142270 brank test summary for BLOCKX.RNG The KS test for those 25 supposed UNI's yields KS p-value= .396534 \$ :: THE BITSTREAM TEST :: The file under test is viewed as a stream of bits. Call them :: :: b1,b2,... Consider an alphabet with two "letters", 0 and 1 :: :: and think of the stream of bits as a succession of 20-letter :: :: "words", overlapping. Thus the first word is blb2...b20, the :: :: second is b2b3...b21, and so on. The bitstream test counts :: :: the number of missing 20-letter (20-bit) words in a string of :: :: 2^21 overlapping 20-letter words. There are 2^20 possible 20 :: :: letter words. For a truly random string of  $2^{21+19}$  bits, the :: :: number of missing words j should be (very close to) normally :: :: distributed with mean 141,909 and sigma 428. Thus :: (j-141909)/428 should be a standard normal variate (z score) :: :: that leads to a uniform [0,1) p value. The test is repeated ::

::

::

:: twenty times. :: THE OVERLAPPING 20-tuples BITSTREAM TEST, 20 BITS PER WORD, N words This test uses  $N=2^{21}$  and samples the bitstream 20 times. No. missing words should average 141909. with sigma=428. \_\_\_\_\_ -.20 sigmas from mean, p-value= .41916 tst no 1: 141822 missing words, tst no 2: 142480 missing words, 1.33 sigmas from mean, p-value= .90879 tst no 3: 141369 missing words, -1.26 sigmas from mean, p-value= .10339 tst no 4: 142381 missing words, 1.10 sigmas from mean, p-value= .86478 tst no 5: 141364 missing words, -1.27 sigmas from mean, p-value= .10131 tst no 6: 142052 missing words, .33 sigmas from mean, p-value= .63056 tst no 7: 141954 missing words, .10 sigmas from mean, p-value= .54156 .10 sigmas from mean, p-value= .54156 -.67 sigmas from mean, p-value= .25250 .33 sigmas from mean, p-value= .62791 -.10 sigmas from mean, p-value= .45968 141624 missing words, tst no 8: 142049 missing words, tst no 9: tst no 10: 141866 missing words, .98 sigmas from mean, p-value= .83717 tst no 11: 142330 missing words, tst no 12: 141971 missing words, .14 sigmas from mean, p-value= .55729 tst no 13: 141785 missing words, -.29 sigmas from mean, p-value= .38572 tst no 14: 141932 missing words, .05 sigmas from mean, p-value= .52112 tst no 15: 141778 missing words, -.31 sigmas from mean, p-value= .37948 tst no 16: 142086 missing words, .41 sigmas from mean, p-value= .66012 tst no 17: 142203 missing words, .69 sigmas from mean, p-value= .75369 tst no 18: 142071 missing words, .38 sigmas from mean, p-value= .64719 tst no 19: 142064 missing words, .36 sigmas from mean, p-value= .64109 tst no 20: 141596 missing words, -.73 sigmas from mean, p-value= .23206

:: The tests OPSO, OQSO and DNA :: :: OPSO means Overlapping-Pairs-Sparse-Occupancy :: :: The OPSO test considers 2-letter words from an alphabet of :: :: 1024 letters. Each letter is determined by a specified ten :: :: bits from a 32-bit integer in the sequence to be tested. OPSO :: :: generates 2^21 (overlapping) 2-letter words (from 2^21+1 :: :: "keystrokes") and counts the number of missing words---that :: :: is 2-letter words which do not appear in the entire sequence. :: :: That count should be very close to normally distributed with :: :: mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should :: :: be a standard normal variable. The OPSO test takes 32 bits at :: :: a time from the test file and uses a designated set of ten :: :: consecutive bits. It then restarts the file for the next de-:: :: signated 10 bits, and so on. :: :: :: :: OQSO means Overlapping-Quadruples-Sparse-Occupancy :: :: The test OQSO is similar, except that it considers 4-letter :: :: words from an alphabet of 32 letters, each letter determined :: :: by a designated string of 5 consecutive bits from the test :: :: :: file, elements of which are assumed 32-bit random integers. :: The mean number of missing words in a sequence of 2^21 four-:: :: letter words, (2^21+3 "keystrokes"), is again 141909, with :: :: sigma = 295. The mean is based on theory; sigma comes from :: :: extensive simulation. :: :: :: The DNA test considers an alphabet of 4 letters:: C,G,A,T,:: :: :: determined by two designated bits in the sequence of random ::

:: integers being tested. It considers 10-letter words, so that :: :: :: as in OPSO and OQSO, there are 2^20 possible words, and the :: mean number of missing words from a string of 2^21 (over-:: lapping) 10-letter words (2<sup>2</sup>1+9 "keystrokes") is 141909. :: :: :: The standard deviation sigma=339 was determined as for OQSO :: :: by simulation. (Sigma for OPSO, 290, is the true value (to :: :: :: three places), not determined by simulation. OPSO test for generator BLOCKX.RNG Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw  $\mathbf{Z}$ р OPSO for BLOCKX.RNG using bits 23 to 32 141947 .130 .5517 OPSO for BLOCKX.RNG using bits 22 to 31 142084 .7265 .602 OPSO for BLOCKX.RNG using bits 21 to 30 141897 -.043 .4830 OPSO for BLOCKX.RNG using bits 20 to 29 142261 1.213 .8874 OPSO for BLOCKX.RNG using bits 19 to 28 142267 1.233 .8913 OPSO for BLOCKX.RNG using bits 18 to 27 142421 1.764 .9612 -.363 OPSO for BLOCKX.RNG using bits 17 to 26 141804 .3582 OPSO for BLOCKX.RNG using bits 16 to 25 141675 -.808 .2095 141330 -1.998 .0229 OPSO for BLOCKX.RNG using bits 15 to 24 OPSO for BLOCKX.RNG using bits 14 to 23 141832 -.267 .3949 OPSO for BLOCKX.RNG using bits 13 to 22 142663 2.599 .9953 OPSO for BLOCKX.RNG using bits 12 to 21 141862 -.163 .4352 142035 .433 OPSO for BLOCKX.RNG using bits 11 to 20 .6676 OPSO for BLOCKX.RNG using bits 10 to 19 -.167 141861 .4338 OPSO for BLOCKX.RNG using bits 9 to 18 -.032 141900 .4872 OPSO for BLOCKX.RNG using bits 8 to 17 142008 .340 .6332 OPSO for BLOCKX.RNG using bits 7 to 16 141849 -.208 .4176 .106 OPSO for BLOCKX.RNG using bits 6 to 15 141940 .5421 OPSO for BLOCKX.RNG using bits 5 to 14 142283 1.289 .9012 .478 OPSO for BLOCKX.RNG .6837 using bits 4 to 13 142048 OPSO for BLOCKX.RNG using bits 3 to 12 141985 .261 .6029 OPSO for BLOCKX.RNG using bits 2 to 11 142502 2.044 .9795 OPSO for BLOCKX.RNG using bits 1 to 10 142461 1.902 .9714 OQSO test for generator BLOCKX.RNG Output: No. missing words (mw), equiv normal variate (z), p-value (p) mw z р using bits 28 to 32 OQSO for BLOCKX.RNG 141916 .023 .5090 using bits 27 to 31 OQSO for BLOCKX.RNG 141831 -.266 .3953 OQSO for BLOCKX.RNG using bits 26 to 30 141878 -.106 .4577 OQSO for BLOCKX.RNG using bits 25 to 29 141984 .253 .5999 OQSO for BLOCKX.RNG using bits 24 to 28 142051 .480 .6845 OQSO for BLOCKX.RNG using bits 23 to 27 142227 1.077 .8592 OQSO for BLOCKX.RNG using bits 22 to 26 142245 1.138 .8724 OQSO for BLOCKX.RNG using bits 21 to 25 141623 -.971 .1659 OQSO for BLOCKX.RNG using bits 20 to 24 141753 -.530 .2981 OQSO for BLOCKX.RNG using bits 19 to 23 141926 .057 .5225 OQSO for BLOCKX.RNG using bits 18 to 22 142284 1.270 .8980

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using bits 17 to 21

using bits 16 to 20

using bits 15 to 19

using bits 14 to 18

using bits 13 to 17

using bits 12 to 16

using bits 11 to 15

using bits 10 to 14

using bits 9 to 13

141982

141837

141976

141937

142048

142120

141810

141709

141526 -1.299

.246

-.245

.226

.094

.470

.714

-.337

-.679

.5973

.0969

.4032

.5894

.5374

.6808

.7624

.3682

.2485

OQSO for BLOCKX.RNG

OQSO for BLOCKX.RNG OQSO for BLOCKX.RNG

OQSO for BLOCKX.RNG

OQSO for BLOCKX.RNG

0QS0 0QS0 0QS0 0QS0 0QS0 0QS0 0QS0 0QS0	for for for for for for	BLOCKX.RNO BLOCKX.RNO BLOCKX.RNO BLOCKX.RNO BLOCKX.RNO BLOCKX.RNO BLOCKX.RNO	G using G using G using G using G using G using G using	bits bits bits bits bits bits bits bits	7 6 5 4 3 2	to to to to to to	11		142069 141930 142300 141768 142161 142234 142048 141934	.070 1.324 479 .853 1.101 .470	.7058 .5279 .9073 .3159 .8032 .8645 .6808 .5333
			or BLOCKX.RN words (mw),		20.01	~~~ T	1	wisto	( - )		1
ομερμε	• 110	. missing v	words (IIIw),	equiv	1101	L IIId -	L Va	Irrate			
DNA DNA DNA DNA DNA DNA DNA DNA DNA DNA	for for for for for for for for for for	BLOCKX . RNG BLOCKX . RNG	G using G using	bits bits bits bits bits bits bits bits	31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8	t t t t t t t t t t t t t t t t t t t	32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12		mw 141759 142159 141374 142117 141506 142022 141802 141749 142024 142114 141733 142277 142187 141506 142206 142153 141699 141958 141513	z 443 .736 -1.579 .613 -1.190 .332 317 473 .338 .604 520 1.085 .819 -1.190 .875 .719	p .3287 .7693 .0572 .7299 .1171 .6302 .3758 .3181 .6324 .7270 .3015 .8609 .7936 .1171 .8093 .7639 .2675 .5571 .1212 .0323 .4926 .2842 .6111 .9544 .5290
		BLOCKX.RNG	-	bits bits		to to	8 7		141934	.073	.5290
		BLOCKX.RNG	~	bits bits		to	6		142024 141677	.338 685	.0324 .2466
		BLOCKX.RN		bits		to	5		141757	449	.3266
		BLOCKX.RN(		bits		to	4		141653	756	.2248
		BLOCKX.RN		bits		to	3		141949	.117	.5466
	-	BLOCKX.RN(	-	bits		to	2		141706	600	.2743

This is the COUNT-THE-1's TEST on a stream of bytes. :: :: :: Consider the file under test as a stream of bytes (four per :: :: 32 bit integer). Each byte can contain from 0 to 8 1's, :: :: with probabilities 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the stream of bytes provide a string of overlapping 5-letter :: :: words, each "letter" taking values A,B,C,D,E. The letters are :: :: determined by the number of 1's in a byte:: 0,1,or 2 yield A,:: :: 3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus :: :: we have a monkey at a typewriter hitting five keys with vari- :: :: ous probabilities (37,56,70,56,37 over 256). There are 5^5 :: :: possible 5-letter words, and from a string of 256,000 (over- ::

:: lapping) 5-letter words, counts are made on the frequencies :: :: for each word. The quadratic form in the weak inverse of :: :: the covariance matrix of the cell counts provides a chisquare :: :: test:: Q5-Q4, the difference of the naive Pearson sums of :: :: (OBS-EXP)<sup>2</sup>/EXP on counts for 5- and 4-letter cell counts. :: Test results for BLOCKX.RNG Chi-square with 5<sup>5</sup>-5<sup>4</sup>=2500 d.of f. for sample size:2560000 chisquare equiv normal p-value Results fo COUNT-THE-1's in successive bytes: byte stream for BLOCKX.RNG 2583.70 1.184 .881726 byte stream for BLOCKX.RNG 2526.01 .368 .643523

This is the COUNT-THE-1's TEST for specific bytes. :: :: :: Consider the file under test as a stream of 32-bit integers. :: :: From each integer, a specific byte is chosen , say the left- :: :: most:: bits 1 to 8. Each byte can contain from 0 to 8 1's, :: :: with probabilitie 1,8,28,56,70,56,28,8,1 over 256. Now let :: :: the specified bytes from successive integers provide a string :: :: of (overlapping) 5-letter words, each "letter" taking values :: :: A,B,C,D,E. The letters are determined by the number of 1's, :: :: in that byte:: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,:: :: and 6,7 or 8 ---> E. Thus we have a monkey at a typewriter :: :: hitting five keys with with various probabilities:: 37,56,70,:: :: 56,37 over 256. There are 5^5 possible 5-letter words, and :: :: from a string of 256,000 (overlapping) 5-letter words, counts :: :: are made on the frequencies for each word. The quadratic form :: :: in the weak inverse of the covariance matrix of the cell :: :: counts provides a chisquare test:: Q5-Q4, the difference of :: :: the naive Pearson sums of (OBS-EXP)^2/EXP on counts for 5-:: :: and 4-letter cell counts. :: Chi-square with 5^5-5^4=2500 d.of f. for sample size: 256000 chisquare equiv normal p value Results for COUNT-THE-1's in specified bytes: bits 1 to 8 2502.04 .029 .511533 bits 2 to 9 2543.23 .611 .729498 bits 3 to 10 1.399 2598.95 .919139 bits 4 to 11 2462.71 -.527 .298976 bits 5 to 12 2589.75 1.269 .897813 bits 6 to 13 2507.29 .103 .541030 bits 7 to 14 2499.43 -.008 .496767 bits 8 to 15 2438.49 -.870 .192185 .295211 bits 9 to 16 2461.94 -.538 bits 10 to 17 2671.80 2.430 .992443 bits 11 to 18 2581.58 1.154 .875694 bits 12 to 19 2528.57 .404 .656910 bits 13 to 20 2536.36 .514 .696426 bits 14 to 21 2474.80 -.356 .360780 bits 15 to 22 -.452 .325607 2468.03 bits 16 to 23 2469.39 -.433 .332570 bits 17 to 24 2621.85 1.723 .957570 bits 18 to 25 2361.68 -1.956 .025222 bits 19 to 26 2501.26 .018 .507093

bits	20	to	27	2389.21	-1.567	.058582
bits	21	to	28	2531.35	.443	.671245
bits	22	to	29	2613.65	1.607	.945997
bits	23	to	30	2400.17	-1.412	.079006
bits	24	to	31	2642.26	2.012	.977883
bits	25	to	32	2495.45	064	.474367

:: THIS IS A PARKING LOT TEST :: :: In a square of side 100, randomly "park" a car---a circle of :: :: radius 1. Then try to park a 2nd, a 3rd, and so on, each :: :: time parking "by ear". That is, if an attempt to park a car :: :: causes a crash with one already parked, try again at a new :: :: random location. (To avoid path problems, consider parking :: :: helicopters rather than cars.) Each attempt leads to either :: :: a crash or a success, the latter followed by an increment to :: :: the list of cars already parked. If we plot n: the number of :: :: attempts, versus k:: the number successfully parked, we get a:: :: curve that should be similar to those provided by a perfect :: :: random number generator. Theory for the behavior of such a :: :: random curve seems beyond reach, and as graphics displays are :: :: not available for this battery of tests, a simple characteriz :: :: ation of the random experiment is used: k, the number of cars :: :: successfully parked after n=12,000 attempts. Simulation shows :: :: that k should average 3523 with sigma 21.9 and is very close :: :: to normally distributed. Thus (k-3523)/21.9 should be a st-:: :: andard normal variable, which, converted to a uniform varia-:: :: ble, provides input to a KSTEST based on a sample of 10. :: CDPARK: result of ten tests on file BLOCKX.RNG Of 12,000 tries, the average no. of successes should be 3523 with sigma=21.9 Successes: 3527 z-score: .183 p-value: .572463 Successes: 3480 z-score: -1.963 p-value: .024796 Successes: 3533 .457 p-value: .676028 z-score: Successes: 3514 -.411 p-value: .340551 z-score: Successes: 3564 1.872 p-value: .969407 z-score: Successes: 3547 z-score: 1.096 p-value: .863437 Successes: 3512 z-score: -.502 p-value: .307734 Successes: 3513 z-score: -.457 p-value: .323972 Successes: 3539 z-score: .731 p-value: .767486 Successes: 3500 z-score: -1.050 p-value: .146807 square size avg. no. parked sample sigma 100. 3522.900 23.039 KSTEST for the above 10: p= .006349

### 

:: should	he (verv	close to)	exponentially distributed with mean ::
			995) should be uniform on [0,1) and ::
			100 values serves as a test of uni- ::
			in the square. Test numbers=0 mod 5 ::
			I is based on the full set of 100 ::
			ints in the 10000x10000 square. ::
			M DISTANCE test
		-	in the file BLOCKX.RNG
Sample no.		avg	equiv uni
5	2.8780	1.1865	.944561
10	.0325	.8289	.032175
15	.4453	.8742	.360819
20	3.0833		.954897
25	.4605	.9374	. 370478
30	.2520	.8370	. 223770
35	1.3678	.8025	.747075
40	.3063	.8304	. 264948
45 50	.9333	.8562 .8187	.608578 .434512
50	.5672 1.4760	.8187	.773133
60	.1485	.7916	.138646
65	3.5344	.7910	.971337
70	.1984	.7681	.180790
78	4.1421	.8526	.984438
80	1.3541	.8643	.743573
85	.3786	.8570	.316505
90	.9284	.8501	.606666
95	1.7096	.8639	.820608
100	1.1507	.8585	.685424
MINIMUM DI	ISTANCE TE	ST for BL	OCKX.RNG
Resul	lt of KS t	est on 20	transformed mindist^2's:
			p-value= .887840
\$\$\$\$\$\$\$\$\$\$\$\$	\$\$\$\$\$\$\$\$\$	\$\$\$\$\$\$\$\$	\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$
::	ТНЕ	3DSPHERE:	S TEST ::
			s in a cube of edge 1000. At each ::
			rge enough to reach the next closest ::
			the smallest such sphere is (very ::
			istributed with mean 120pi/3. Thus ::
			ential with mean 30. (The mean is ::
			ulation). The 3DSPHERES test gener- ::
:: ates 40	)00 such s	pheres 20	times. Each min radius cubed leads ::
:: to a ur	niform var	iable by n	means of $1-\exp(-r^3/30.)$ , then a ::
:: KSTESI	] is done	on the 20	p-values. ::
:::::::::			
	The 3DSPH	ERES test	for file BLOCKX.RNG
sample no: 1	r^3=	4.664	p-value= .14398
sample no: 2	r^3=	21.443	p-value= .51070
sample no: 3	r^3=	70.671	p-value= .90517
sample no: 4	r^3=	16.043	p-value= .41420
sample no: 5	r^3=	8.593	p-value= .24907
sample no: 6	r^3=	13.257	p-value= .35720
sample no: 7	r^3=	14.018	p-value= .37329

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p-value= .00741

r^3= .223

sample no: 8

<pre>sample no: 9 sample no: 10 sample no: 11 sample no: 12 sample no: 13 sample no: 14 sample no: 14 sample no: 16 sample no: 16 sample no: 17 sample no: 18 sample no: 19 sample no: 20 A KS test is</pre>	$r^{3} = 52$ $r^{3} = 4$ $r^{3} = 22$ $r^{3} = 59$ $r^{3} = 184$ $r^{3} = 56$ $r^{3} = 82$ $r^{3} = 15$ $r^{3} = 15$ $r^{3} = 51$ $r^{3} = 30$ $r^{3} = 39$	088         p-valu           046         p-valu           994         p-valu           855         p-valu           201         p-valu           673         p-valu           638         p-valu           210         p-valu           278         p-valu           978         p-valu           468         p-valu	he= .71699 he= .82382 he= .12617 he= .53534 he= .86401 he= .99785 he= .93637 he= .39771 he= .81779 he= .63781 he= .73459 es.	
			p-value= .5525 \$\$\$\$\$\$\$\$\$\$\$	81
	iiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiiii			:::::
	~		et uniforms on [0,1). Sta	
			test finds j, the number	
			to 1, using the reductio	
			by floating integers from	
			are found 100,000 times,	
			nes j was <=6,7,,47,>=	
			test for cell frequencie	
				:::::
		ZE TEST FOR BL ed frequency c		
	p)/sqrt(exp)		Jouries	
		<=6,7,8,,47	7,>=48:	
-1.5 -1	2 -1 1	2 1	3	
1 -	.1 .6	1 7 9	2 0	
	.0 1.0	/ .2		
		.25	.3	
	.91	.25 .3 .1		
.6 -	.6 -1.1 .2 -1.2	.3 .1 1 3.1	1 .0	
2	.2 -1.2	1 5.1	.0	
	-square with 4	2 degrees of f	Freedom: 42.931	
		2 p-value= .5		

## \$

:: The OVERLAPPING SUMS test :: :: Integers are floated to get a sequence U(1),U(2),... of uni-:: :: form [0,1) variables. Then overlapping sums, :: S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed. :: :: :: The S's are virtually normal with a certain covariance mat-:: :: rix. A linear transformation of the S's converts them to a :: :: sequence of independent standard normals, which are converted :: :: to uniform variables for a KSTEST. The p-values from ten :: :: KSTESTs are given still another KSTEST. :: Test no. 1 p-value .341673

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	Test	no.	2		p-value	.965883	
	Test	no.	3		p-value	.301137	
	Test	no.	4		p-value	.580212	
	Test	no.	5		p-value	.089571	
	Test	no.	б		p-value	.602217	
	Test	no.	7		p-value	.390491	
	Test	no.	8		p-value	.086171	
	Test	no.	9		p-value	.956151	
	Test	no.	10		p-value	.002479	
f	the OSU	SUM test for BLOCKX.RNG					

KSTEST on the above 10 p-values: .646815

Results o

This is the RUNS test. It counts runs up, and runs down, :: :: :: in a sequence of uniform [0,1) variables, obtained by float-:: :: ing the 32-bit integers in the specified file. This example :: :: shows how runs are counted: .123,.357,.789,.425,.224,.416,.95:: :: contains an up-run of length 3, a down-run of length 2 and an :: :: up-run of (at least) 2, depending on the next values. The :: :: covariance matrices for the runs-up and runs-down are well :: :: known, leading to chisquare tests for quadratic forms in the :: :: weak inverses of the covariance matrices. Runs are counted :: :: for sequences of length 10,000. This is done ten times. Then :: :: repeated. :: The RUNS test for file BLOCKX.RNG

:

Up and down runs in a sample of 10000

Run test for BLOCKX.RNG runs up; ks test for 10 p's: .306738 runs down; ks test for 10 p's: .740152 Run test for BLOCKX.RNG runs up; ks test for 10 p's: .004499 runs down; ks test for 10 p's: .155079

#### 

:: This is the CRAPS TEST. It plays 200,000 games of craps, finds:: :: the number of wins and the number of throws necessary to end :: :: each game. The number of wins should be (very close to) a :: :: normal with mean 200000p and variance 200000p(1-p), with :: :: p=244/495. Throws necessary to complete the game can vary :: :: from 1 to infinity, but counts for all>21 are lumped with 21. :: :: A chi-square test is made on the no.-of-throws cell counts. :: :: Each 32-bit integer from the test file provides the value for :: :: the throw of a die, by floating to [0,1), multiplying by 6 :: :: and taking 1 plus the integer part of the result. :: Results of craps test for BLOCKX.RNG No. of wins: Observed Expected 98585.86 98771 98771= No. of wins, z-score= .828 pvalue= .79618

Analysis of Throws-per-Game: Chisq= 17.12 for 20 degrees of freedom, p= .35474

Thro	ws O	Observed 1			ected	(	Chisq		Sum	L
1		66881		66666.7			.689		.6	89
2		37413		37654.3			1.547		2.2	36
3	3	27170		26954.7			1.719		3.9	55
4	Ł	19296		19313.5			.016		3.9	71
Ę		13752		13851.4			.714		4.6	84
e		9843		9943.5			1.017		5.7	01
7	,	7128			45.0		.041		5.7	41
8	3	5151			.39.1		.028		5.7	69
ç	3616			3699.9			1.901		7.6	70
10	)	2659			66.3		.020		7.6	90
11	-	1929			23.3		.017		7.7	07
12	2	1378			888.7		.083		7.7	90
13	3	1007			03.7		.011		7.8	01
14	14 790			726.1			5.616		13.4	17
15	15 539			5	525.8		.330		13.7	46
16	16 394			3	881.2		.433		14.1	79
17	17 294			2	276.5		1.102		15.2	82
18	3	206			200.8		.133		15.4	15
19	)	153			46.0		.337		15.7	52
20	20 118			1	.06.2		1.308		17.0	60
21	21 283			2	287.1		.059		17.1	19
SUMMARY FOR BLOCKX.RNG										
p-v	ralue	for n	ο.	of	wins	:	.796183	1		
	-	-	-					-		

p-value for throws/game: .354742

# 

Results of DIEHARD battery of tests sent to file ZZREPORT.TXT